Instructions

Please complete all **4** problems attached.

Problem 1. Linear Algebra

Consider the following matrices

$$A = \begin{pmatrix} 3 & 4 & 4 \\ 1 & a & 2 \\ 2 & 3 & 3 \end{pmatrix} \text{ and } B = \begin{pmatrix} 2 & -1 & 0 \\ -5 & 3 & -1 \\ 0 & 1 & a \end{pmatrix} \text{ where } a \in R$$

1. Find $\det A$ and $\det B$
2. Find $\det(AB)$ and $\det\left((A^T B)^4\right)$
3. Find all a values, for which A is nonsingular, and then compute for these values a the determinant $\det(A^{-1})$
4. Determine 3 eigenvalues of $(A - A^T)$
5. Determine **<u>any one</u>** of the three eigenvectors of $(A - A^T)$

Problem 2. Vector Calculus

$\mathbf{F}$ is a continuous force field on $\mathbb{R}^2$ given by

$$\mathbf{F}(x,y) = x^2\mathbf{i} - xy\mathbf{j}\,, \tag{1}$$

and $\mathbf{r}$ is a smooth curve C given by

$$\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j}\,, \tag{2}$$

bounded by the parameter interval

$$0 \le t \le \pi/2\,. \tag{3}$$

Note that in general, C is given by the vector equation

$$\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j}. \tag{4}$$

Questions:

1. Draw the force field given by Equation (1).
2. Draw the curve C given by Equation (2) for the given parameter interval.
3. Compute the work W done by this force $\mathbf{F}$ in moving a particle along the curve C, noting that work can be computing using the line integral of $\mathbf{F}$ along C as given by

$$W = \int_C \mathbf{F}\cdot d\mathbf{r} = \int_a^b \mathbf{F}(\mathbf{r}(t))\cdot \mathbf{r}'(t)\,dt\,.$$

4. If the work done is negative, what does that imply?

Problem 3. Differential equation

Consider the differential equation

$$(3x^2 - 2y^2)dx + (1 - 4xy)dy = 0$$

a) Is this differential equation exact? Explain your reasoning.
b) Find the solution to this differential equation.

Problem 4. Numerical Analysis

Given the first-order transient advection equation $\partial u/\partial t + \partial u/\partial x = 0$. Consider 3 different stencils (see below) for its finite-difference solution. For each stencil (where τ and h denote the step size in time t and space x, respectively)

(1) develop a finite-difference approximation of the equation,

(2) define an order of approximation with respect to time, t, and space, x, variables,

(3) use the finite-difference Fourier analysis to develop the criteria for stability of each approximation.