GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff

School of Mechanical Engineering

Ph.D. Qualifiers Exam - Spring Semester 2020

APPLIED MATH

EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

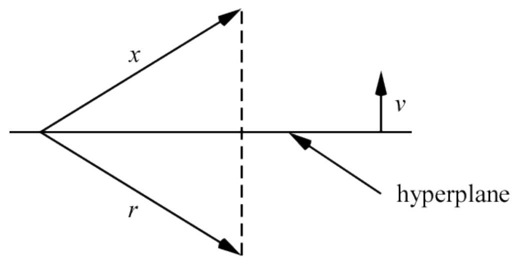
✷ Please sign your **name** on the back of this page —

APPLIED MATH WRITTEN EXAM

SPRING 2020

1) Answer the following questions:

One useful transformation matrix is the Householder reflection matrix P. The idea behind this matrix is shown in the figure to the right. Here, a hyperplane is defined by a unit normal vector v. The vector x is reflected through this hyperplane into the vector r using the matrix P. So, $Px = r$, where $P = I - 2vv^T$ and I is the identity matrix. Note as well that $|x| = |r|$.



a) Derive P.

b) Show that P is orthonormal.

The Householder reflection matrix can be used to find the QR decomposition of a matrix given by $A = QR$, where Q is an orthonormal matrix and R is a right (upper) triangular matrix.

c) Show how this decomposition can be used to find the inverse of the matrix A.

2) Answer the following question:

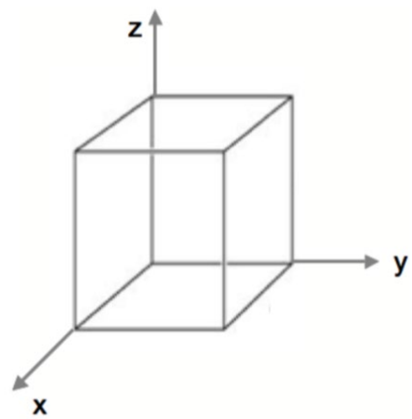
Given the following equation: $2\frac{d^2y}{dx^2} + 3y + 7 = 0$, for $-\infty < x < +\infty$

While we assume that we know a value for y at x = 0, find a series solution for y in powers of x.

3) Answer the following questions:

a.) Given the vector field $\vec{g} = \left(x^2 y\,\hat{i} + \left(y^3 - 3x\right)\hat{j} + 4z^2\,\hat{k}\right)$, determine the divergence and curl of $\vec{g}$. Determine the locus of points in the $z = 0$ plane for which the divergence of this vector field vanishes.

b.) Evaluate the flux of the vector field $\vec{F} = \exp(2x)\,\hat{i} + y\,\hat{j}$ over the surface of the unit cube shown below with corners at (0,0,0) and (1,1,1). Next, evaluate the flux through the surface of the unit cube if it is instead centered at the origin.



4) For $\frac{d^2\theta}{dt^2} + \frac{g}{l}\sin\theta = 0$ with $\theta(0) = 0.5$. Solve based on Euler's method with a step size of 0.5 and write the problem as a system of first order ODEs and give the complete solution at t = 2.