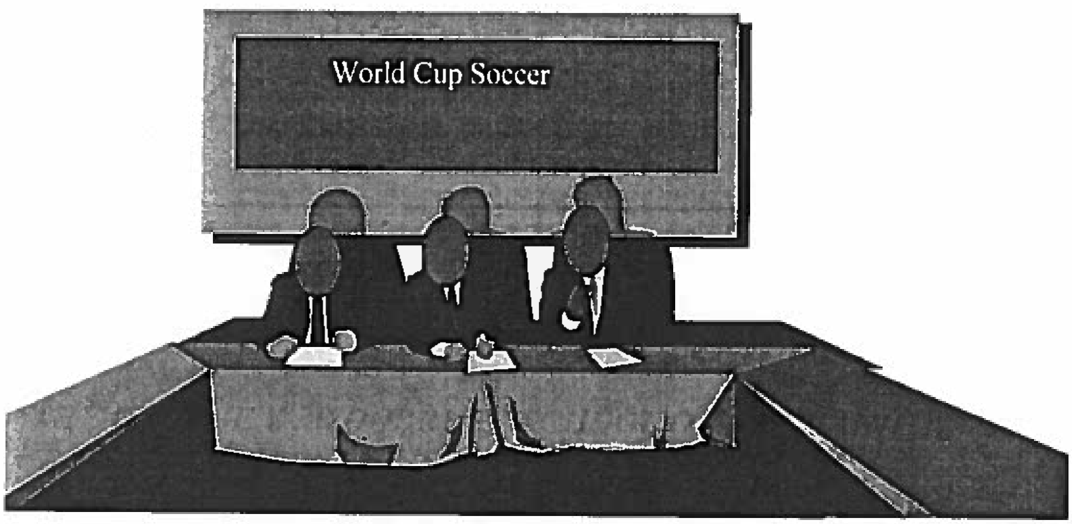
COMPUTER-AIDED ENGINEERING

Ph.D. QUALIFIER EXAM – Fall 2014

THE GEORGE W. WOODRUFF SCHOOL OF MECHANICAL ENG.
GEORGIA INSTITUTE OF TECHNOLOGY
ATLANTA, GA 30332-0405

S.-K. Choi, C. Paredis, D. Rosen, S. Sitaraman (Chair), and Y. Wang



- All questions in this exam have a common theme: *World Cup Soccer*
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- *During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.*

GOOD LUCK!

Question 1 - Geometric Modeling

The path of a soccer ball will be modeled in this problem. A player is running down the field with the ball and kicks the ball at $\mathbf{p}_0 = (30, 30, 0)$ with an initial velocity of $\mathbf{p}_0^u = (-20, -8, 10)$. When the ball reaches the point $\mathbf{p}_1 = (15, 22, 3)$, the ball is traveling with a velocity of $\mathbf{p}_1^u = (-16, -16, 0)$. At that time, a gust of wind starts blowing which causes the ball to curve even more such that it ends up at point $\mathbf{p}_2 = (-1, -1, 1)$ with a velocity of $\mathbf{p}_2^u = (-10, -15, -1.5)$.

A schematic of the kick is shown below in the XZ plane. Note that the ball travels in 3 dimensions. We will model the path of the ball as a composite cubic Hermite curve. Equations for a Hermite curve are given at the bottom of the page.

Answer the following questions:

a) Derive the equation for the first Hermite curve (from $\mathbf{p}_0$ to $\mathbf{p}_1$). Simplify the equations into the form: $\mathbf{a}_3 u^3 + \mathbf{a}_2 u^2 + \mathbf{a}_1 u + \mathbf{a}_0 = \mathbf{k}(u)$

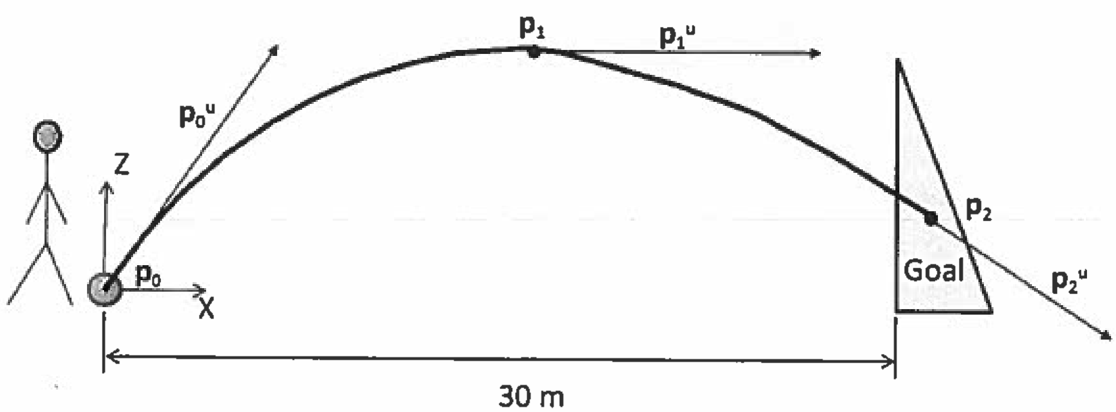
b) Compute the point on this first curve at $u = 0.3$.

c) Derive the equation of the second Hermite curve (from $\mathbf{p}_1$ to $\mathbf{p}_2$). You do NOT need to simplify this equation.

d) Compute the point on this second curve at $u = 0.5$.

e) Sketch the path of the ball in the XY and XZ planes from $\mathbf{p}_0$ to $\mathbf{p}_2$. Include the tangent vectors in both graphs and plot the points you computed in parts b) and d). Did you compute the point coordinates correctly?

f) You are to compute the Y and Z coordinates of the point where the ball goes into the net (that is, where it passes the X = 0 plane). Describe the procedure that you will use to compute the point's coordinates. Then, perform the procedure and show your work.

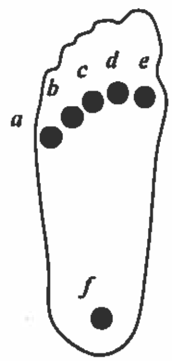


$$p(u) = F_0(u)p_0 + F_1(u)p_1 + F_2(u)p_0^u + F_3(u)p_1^u$$
$$F_0(u) = 2u^3 - 3u^2 + 1$$
$$F_1(u) = -2u^3 + 3u^2$$
$$F_2(u) = u^3 - 2u^2 + u$$
$$F_3(u) = u^3 - u^2$$

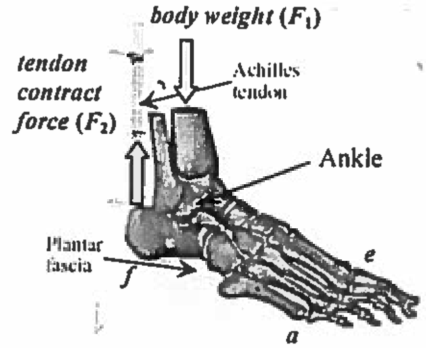
Question 2 - Finite-Element Analysis

As a design engineer, you are assigned a task to design the customized insole for an athlete. Prior study has shown that the stress concentrations are at the six spots (*a,b,…,f*) when the athlete stands on the ground, denoted by the circles in Figure-A on the right.

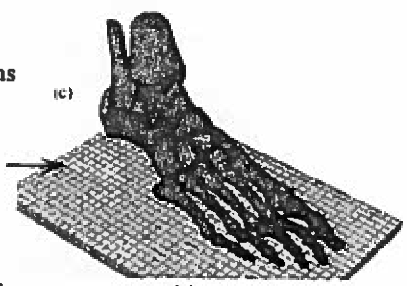
The foot and ankle can be modeled as in Figure-B. The body weight (F_1) is applied vertically to the ankle. The Achilles tendon contracts to maintain the balance with the contract force (F_2). The muscles of *plantar fascia* connecting the heel and the five bones are modeled by 5 truss elements to support tensile stress.



(A) six stress concentration areas



(B) tissue-bone structure of foot



(C) insole serves as a cushion

a) Based on the model in Figure-B with the given external loads, and assuming the six spots (*a,b,…,f*) touch the ground, build a simplified finite-element model to estimate the deformation at the ankle area in Figure-A. Make necessary assumptions of lengths, cross-section areas, materials, etc.
b) Build a second finite-element model in order to design the thickness of insole (elastic and flexible) that serves as the cushion between the stress concentration areas and the ground. Estimate the deformation at the ankle area again.

1. State all of your assumptions clearly.
2. Show all of your calculations.
3. Show the boundary conditions and loading conditions.
4. Write down the element stiffness matrix and assembly stiffness matrix.

Element A - Stiffness Matrix

$$[K]=\frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

where *E, A,* and *L* are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; $l=(x_2-x_1)/L$ and $m=(y_2-y_1)/L$ are directional *cos*() and *sin*() respectively.

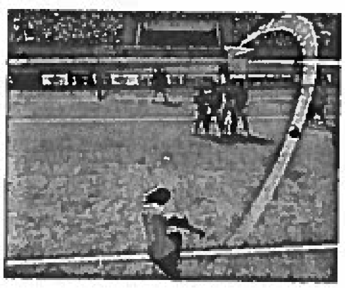
Element B - Stiffness Matrix

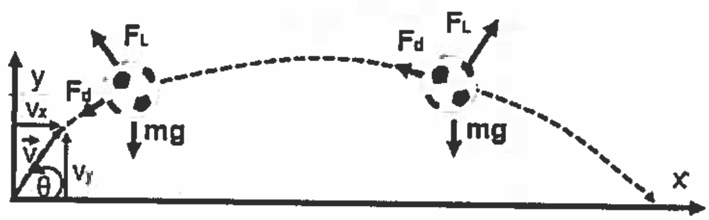
$$[K]=\frac{EI}{L^3}\begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix}$$

where *E, I,* and *L* are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

Question 3 – Numerical Analysis

A soccer player hits the ball right on the little air nozzle (valve/adapter) of the ball, or he hits in the middle of the ball and tries to put topspin on it; very similar to tennis players. Obviously, one of the key components of controlling the ball trajectory is spinning the ball. The simulation of the soccer ball trajectory can be represented by two forces: the lift force (F_L) and the drag force (F_d). The lift force due to spin is also called the magnus force and it always acts at right angles to the drag force (F_d) and to the spin axis. The drag force $F_d = \frac{1}{2}\rho C_d A v^2$ acts backwards and the lift force $F_L = \rho C_L D^3 f v$ acts upwards and at right angles to the path of the ball. In this problem, the lift coefficient C_L=1.23, the drag coefficient C_d=0.1, the density of the air $\rho = 1.2\ kg/m^3$, the diameter of the ball D=0.22 m, the mass of the ball m=0.43 kg, and the gravity of the acceleration g=9.8 m/s^2. Also, f, A, Θ, and v denote the spin frequency, the cross-sectional area, the launch angle, and the velocity of the ball, respectively. Assume that the soccer player is trying to swing the ball at 25 m from the goal. He struck the ball with a velocity of 25 m/s with a launch angle of 30° in such a way as to cause it to spin at 10 rev/s.





(a) Obtain the equations of motion for the soccer ball trajectory. Write down two first-order differential equations in terms of the velocity in the horizontal direction (v_x) and the velocity in the vertical direction (v_y) as shown in the figure.

(b) Determine approximate values of the velocity components at the time t=0.2 sec with a step size of h=0.1.

(c) Now, determine the distance the ball travels, x(t) and y(t), based on the information obtained from (b).

(d) With the numerical methods you selected, make comments on the anticipated behavior of the global and local errors of the given problem as t increases. In addition, do you expect to have significant accuracy improvements as smaller and smaller step sizes are used?