# *COMPUTER-AIDED ENGINEERING*

## *Ph.D. QUALIFIER EXAM – Spring 2014*

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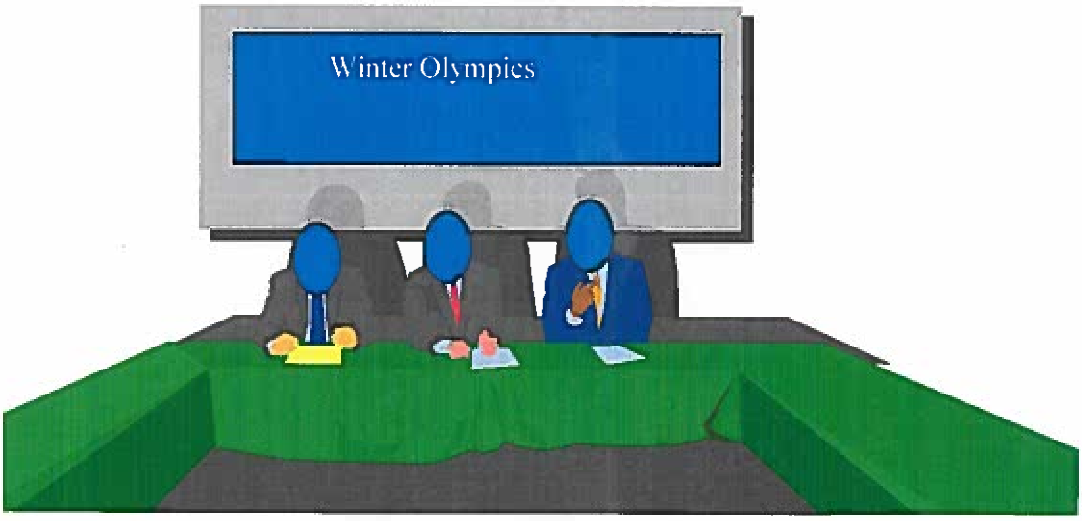
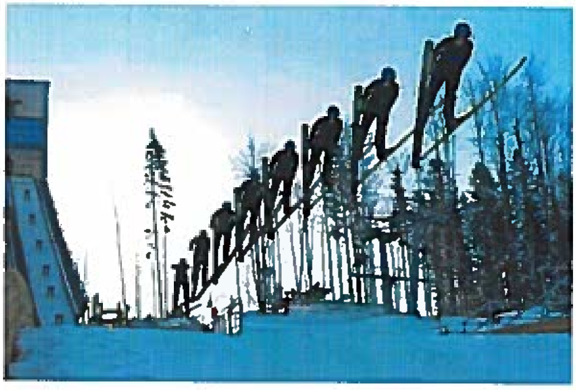



- All questions in this exam have a common theme: ***Winter Olympics***
- Answer all questions.
- Make suitable assumptions when data is not available or when you do not follow a question. State your assumptions clearly and justify.
- Show all steps and calculations.
- ***During ORALS, you will be given an opportunity to tell us how CAE fits into your doctoral research. Please come prepared to make this opening statement.***

***GOOD LUCK!***

## Question 1 - Geometric Modeling



This problem concerns a flight of a ski jumper at the Winter Olympics. Assume that the path followed by the jumper consists of three curve segments: one for the jump, one for his flight, and one after landing. See the schematic below.

Control vertices for the curve that models flight are: $\mathbf{p}_0 = (220, 70)$, $\mathbf{p}_1 = (170, 62)$, $\mathbf{p}_2 = (150, 60)$, $\mathbf{p}_3 = (100, 30)$. Control vertices for the curve that models the jump are: $\mathbf{q}_0 = (270, 130)$, $\mathbf{q}_1 = (256, 80)$, $\mathbf{q}_2 = (235, 72.4)$, $\mathbf{q}_3 = (220, 70)$. Assume the units are meters.

Answer the following questions:

a) Derive the equation for the cubic Bezier curve that corresponds to the given control vertices. Simplify the equations into the form: $\mathbf{a}_3 u^3 + \mathbf{a}_2 u^2 + \mathbf{a}_1 u + \mathbf{a}_0 = \mathbf{k}(u)$

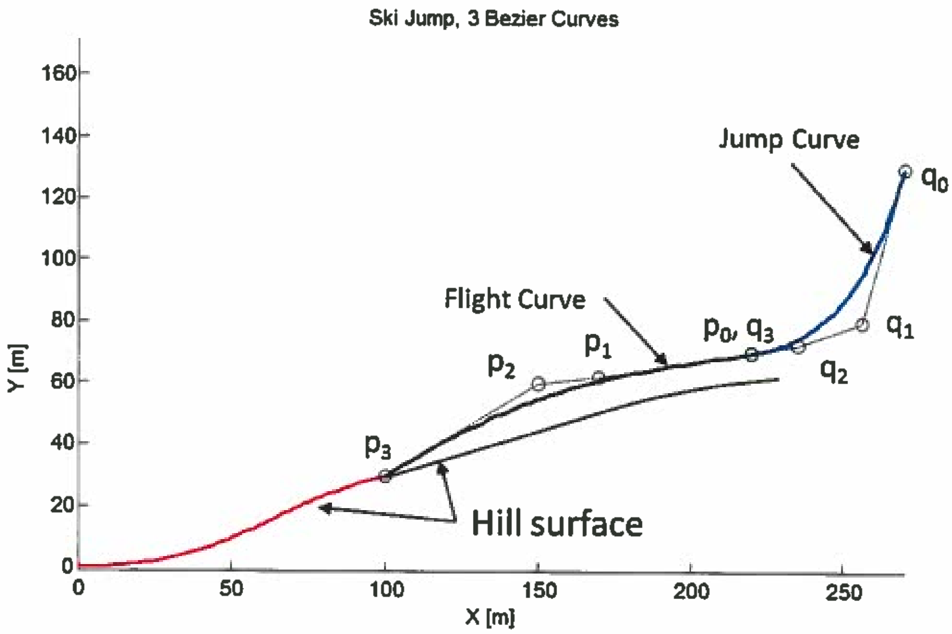
b) Compute the point on the curve at $u = 0.7$.

c) Derive or state the relationship between Bezier curve CVs and Hermite curve tangent vectors. Compute the tangent vectors at the end of the jump curve ($\mathbf{q}_2, \mathbf{q}_3$) and the beginning of the flight curve ($\mathbf{p}_0, \mathbf{p}_1$).

d) What is the continuity condition between the jump and flight curves? Justify your answer quantitatively (i.e., with calculations).

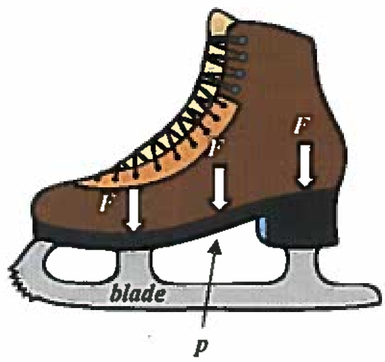
e) Compute the CVs on the flight curve for $C^1$ continuity between the jump and flight curves.

f) Does $C^1$ continuity make sense for this situation? Justify your answer with reference to the relevant physics of the jump.



**Question 2 – Finite-Element Analysis**

As a design engineer for an ice skate manufacturer, you are assigned to design a shoe, as shown in the figure on the right.




a) Assume that the external load can be simplified as three forces $F$'s, as shown. Build two finite-element models for the structural integrity analysis at the portion(s) of the shoe where fractures are most likely to occur:
   i. For the first model, assume the blade to be rigid.
   ii. For the second model, assume that the blade is not rigid.

b) From your models, formulate an assembled stiffness matrix to determine the deformation at the point $p$. Make necessary assumptions of lengths, cross-section areas, materials, etc.

***Element A - Stiffness Matrix***

$$[K] = \frac{EA}{L}\begin{bmatrix} l^2 & lm & -l^2 & -lm \\ lm & m^2 & -lm & -m^2 \\ -l^2 & -lm & l^2 & lm \\ -lm & -m^2 & lm & m^2 \end{bmatrix}$$

where $E$, $A$, and $L$ are the Modulus of Elasticity, Area of cross-section, and Length of the element respectively; $l=(x_2-x_1)/L$ and $m=(y_2-y_1)/L$ are directional $cos()$ and $sin()$ respectively.

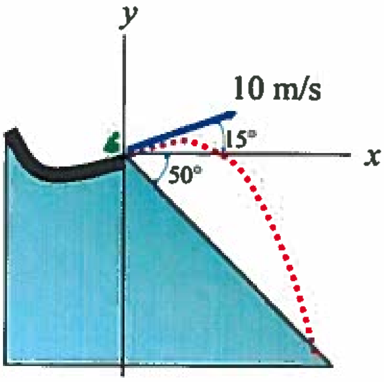
***Element B - Stiffness Matrix***

$$[K] = \frac{EI}{L^3}\begin{bmatrix} 12 & -6L & -12 & -6L \\ -6L & 4L^2 & 6L & 2L^2 \\ -12 & 6L & 12 & 6L \\ -6L & 2L^2 & 6L & 4L^2 \end{bmatrix}$$

where $E$, $I$, and $L$ are the Modulus of Elasticity, Moment of inertia, and Length of the element respectively;

## Question 3 – Numerical Analysis

You are asked to design a safe landing surface for a ski jump at the 2014 Sochi Winter Olympics. For the safe landing surface, the primary requirement is to accurately estimate a skier's flight path. As shown in the figure, the angle of the takeoff ramp is 15° ($\alpha$=15°) above the horizontal and the landing slope is inclined at 50° ($\beta$=50°). A 70 kg skier leaves the ramp with a velocity of 10 m/s. The gravitational acceleration $g$ is 9.8 m/s$^2$. Assume that the ramp is frictionless and the air resistance is negligible (neglect aerodynamic forces).




**(a)** Develop a projectile equation and determine the distance from the takeoff ramp to where the ski jumper lands, measured along the sloped surface direction.

The velocity in the $x$- and $y$-direction has been monitored several times during the jump and it is summarized in the table below.

| $t$ (sec) | 0.00 | 0.10 | 0.20 | 0.32 | 0.36 | 0.40 | 0.44 | 0.54 | 0.64 | 0.70 | 0.80 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $v_x$ (m/s) | 0.00 | 0.97 | 1.93 | 3.09 | 3.48 | 3.86 | 4.25 | 5.22 | 6.18 | 6.76 | 7.73 |
| $v_y$ (m/s) | 2.59 | 1.61 | 0.63 | -0.55 | -0.94 | -1.33 | -1.72 | -2.70 | -3.68 | -4.27 | -5.25 |

**(b)** Without actually solving the problem, suggest the best strategy for attaining the highest accuracy of the estimate of the flying distance with the given data in the table.

**(c)** Determine the $y$-location of the jumper at $t$=0.44 sec using your suggested strategy. Comment on the estimation accuracy compared to the answer using the projectile equation in (a).

**(d)** Assuming that additional data points are obtained in the middle of each time interval in the table above, how would the accuracy of your estimate change? Provide a detailed explanation.