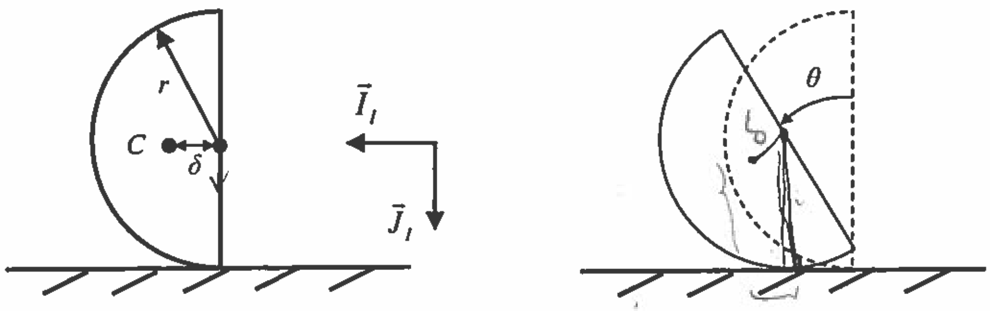
Dynamics and Vibrations Ph.D. Qualifying Exam
Spring 2015

Instructions:
Please work 3 of the 4 problems on this exam. <u>It is important that you clearly mark which three problems you wish to have graded</u>. For the three problems that you select, show all your work in order to receive proper credit. You are allowed to use a calculator.

Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1.

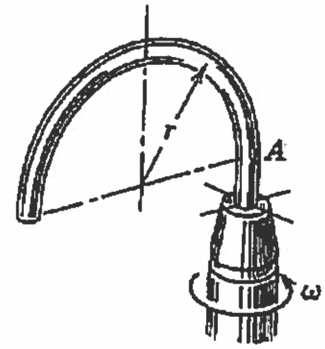
The semicylinder of mass m shown below is release from rest, and there is enough friction to prevent slipping throughout the ensuing motion. The mass center is shown as point C, and let the moment of inertia of the object about C in the $\vec{K}_I$ direction be given by I^C_{zz}. Furthermore, let the position of point C be given as $\vec{r}_{O \to C} = x\vec{I}_I + y\vec{J}_I$.



a) Consider the states of the body as x, y, and θ. Write the three differential equations of motion of the body (good at any angle θ) in terms of gravity, friction, and normal forces.
b) Write a single second-order differential equation governing θ strictly in terms of $\theta, \dot{\theta}, \ddot{\theta}$, and other constants (i.e., not including normal force or friction terms).
c) Find the velocity (vector) of the center of mass C with respect to frame I when $\theta = 90$ deg if the object is released from rest in the vertical position.

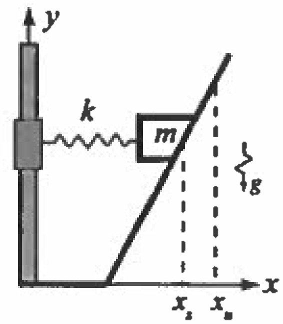
Problem 2.

Determine the bending moment M at the tangency point A in the semi-circular rod of radius r and mass m as it rotates about the tangent axis with a constant angular velocity ω. Neglect the moment mgr produced by the weight compared with that caused by the rotation of the rod.



Problem 3.

A mass m attached to a linear spring of constant k slides along the frictionless inclined surface that follows the equation $y=bx-c$ as shown in the figure. The spring is attached to a massless collar that can slide vertically on the pole without friction. Assume that the spring is unstretched at x_u.



a) Determine the position x_s where the system is in static equilibrium.

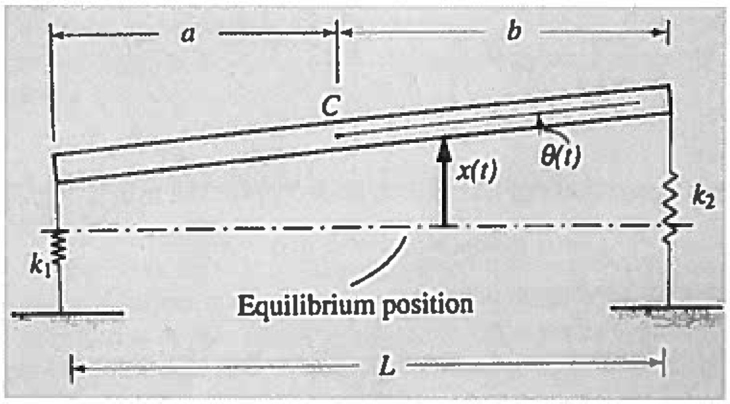
b) Derive the equation of motion in terms of coordinate x.

c) Detemine the equation of motion for small motions $\hat{x}$ about the equilibrium position x_s.

You may assume that the spring is always horizontal and that the mass never strikes the floor.

Problem 4.

Consider the following 2-DOF simplified automobile model that consists of a rigid slab supported on two springs. The slab mass is m and its mass moment of inertia about the centroid (point C) is I_C. The translational and rotational displacements x and θ are measured from the horizontal static equilibrium position.



(a) Derive the equations of motion (in terms of x and θ) for free vibrations symbolically, and express them in the matrix form.

(b) Given $m = 1500$ kg, $I_C = 2000$ kg.m^2, $k_1 = 36$ kN/m, $k_2 = 40$ kN/m, $a = 1.3$ m, and $b = 1.7$ m. Calculate the natural frequencies and mode shapes. Sketch the mode shapes.

(c) For the numerical data in part (b), find the response to the initial conditions $x(0) = 0.1\,\text{m}$, $\theta(0) = 0.291\,\text{rad}$.

(d) Is the response in part (c) periodic? If so, calculate its period.