Dynamics and Vibrations Ph.D. Qualifying Exam
Fall 2015

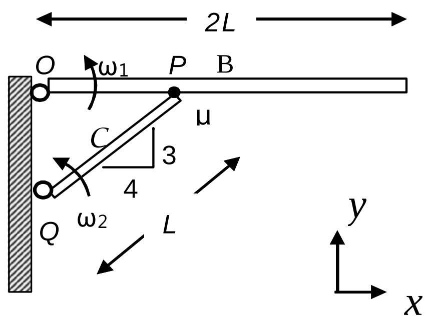
Instructions:
Please work 3 of the 4 problems on this exam. It is important that you clearly mark which three problems you wish to have graded. For the three problems that you select, show all your work in order to receive proper credit. You are allowed to use a calculator.

Be sure to budget your time; concentrate on setting up the problem solution first and leave algebra until the end. When necessary, you may leave your answers in terms of unevaluated numerical expressions. Good Luck!

Problem 1.

Trap door B of mass m opens at a constant angular velocity ω_1 by rotating a light strut C which slides along the door at contact point P. The door and strut are pinned to the wall as shown. For the instant shown, answer the following parts and state any simplifying assumptions if needed.

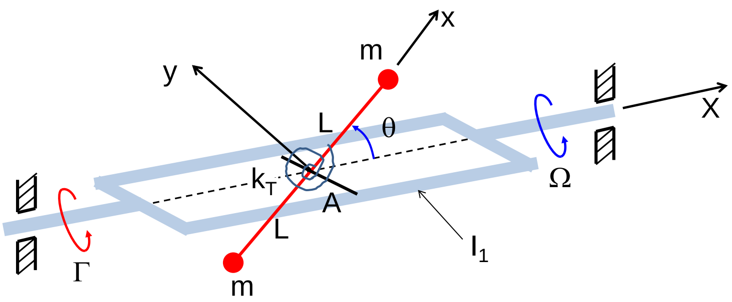
a) Determine the strut angular velocity ω_2.
b) Determine the contact force f applied by the strut to the door.
c) Determine the torque that must be applied by a motor at Q.



Problem 2.

A gimbal spins about an inertially-fixed X-axis with angular velocity Ω. The gimbal has mass m_1 and has a mass moment of inertia of I_1 about the X-axis. A dumbbell turns relative to the gimbal so that the angle θ is variable. The dumbbell consists of 2 masses m connected by a rigid, massless arm of length 2L. The turning of the dumbbell relative to the gimbal is resisted by a torsional spring of stiffness k_T, but is otherwise free to turn about the body-fixed z-axis. The spring is undeformed when $\theta = 0$.

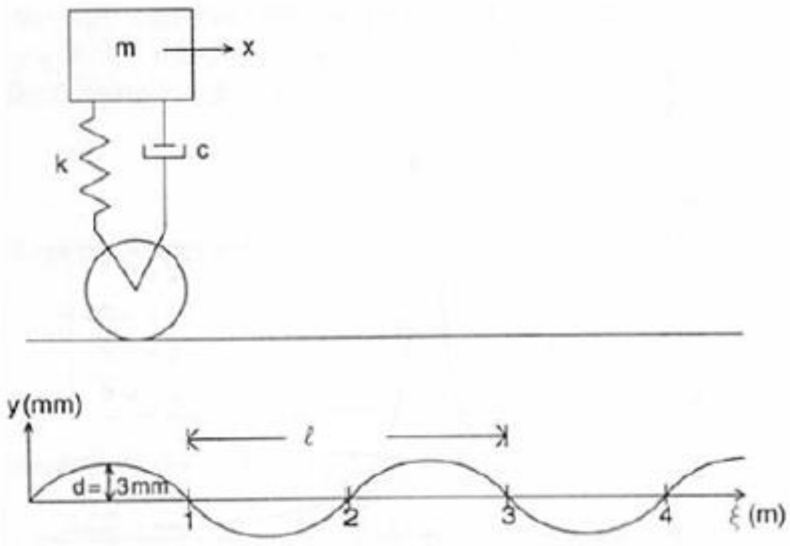
a) Assume that a torque Γ applied to the gimbal about the X-axis keeps the rotation rate Ω = const. For that case, find a differential equation for the coordinate $\theta(t)$ and give an expression for the torque necessary to maintain the constant rotation rate.
b) For the case where the driving torque is zero, state all conservation laws that apply, and give expressions for those relations in terms of m, L, I_1, k_T, Ω, and θ.



Problem 3.

A simplified model of a vehicle and its suspension system is illustrated in the figure. The vehicle travels with a constant horizontal velocity *v* over a road of sinusoidal contour.

a) Assuming no damping (and only assume no damping in this sub-question), identify the speed at which resonance occurs.
b) Determine the amplitude response of the vehicle in terms of the velocity *v* and the other system parameters given in the figure. Feel free to define new variables as needed (e.g., frequency ratio *r*, damping ratio ζ).
c) Determine the amplitude of the force developed in the spring in terms of the velocity *v* and other system parameters.
d) If the damping ratio of the suspension is chosen to be approximately 1.0, as is common in suspension design, discuss the vertical acceleration of the vehicle (and thus occupancy comfort) as the vehicle speed increases well past resonance (i.e., a speed corresponding to 3X or 4X times the resonant frequency). How does this acceleration compare to that experienced in an undamped suspension?



Problem 4.

Consider small oscillations of the double pendulum shown below, i.e. θ_1 and θ_2 are small. There is no friction at the pivots, no air drag, i.e. the system is undamped, and the links are rigid.

a) The linearized equations of motion are
$$(m_1+m_2)L_1\ddot{\theta}_1+m_2L_2\ddot{\theta}_2+(m_1+m_2)g\theta_1=0,$$
$$m_2L_1\ddot{\theta}_1+m_2L_2\ddot{\theta}_2+m_2g\theta_2=0.$$
Clearly explain your methodology and steps to obtain these governing equations.

b) Let $m_1=m_2=m$ and $L_1=L_2=L$. Find the natural frequencies and mode shapes. Sketch the mode shapes.

c) For the parameters in part b) (i.e. $m_1=m_2=m$ and $L_1=L_2=L$), obtain the response to initial conditions $\begin{Bmatrix}\theta_1(0)\\ \theta_2(0)\end{Bmatrix}=\begin{Bmatrix}0.1\\ -0.1414\end{Bmatrix}$ rad, $\begin{Bmatrix}\dot{\theta}_1(0)\\ \dot{\theta}_2(0)\end{Bmatrix}=\begin{Bmatrix}0\\ 0\end{Bmatrix}$.

d) Consider the system at rest (in static equilibrium). Mass m_1 is excited horizontally by a harmonic force at frequency ω. Find the non-zero frequency at which the forced response of m_1 is zero. Neglect the initial condition effects.