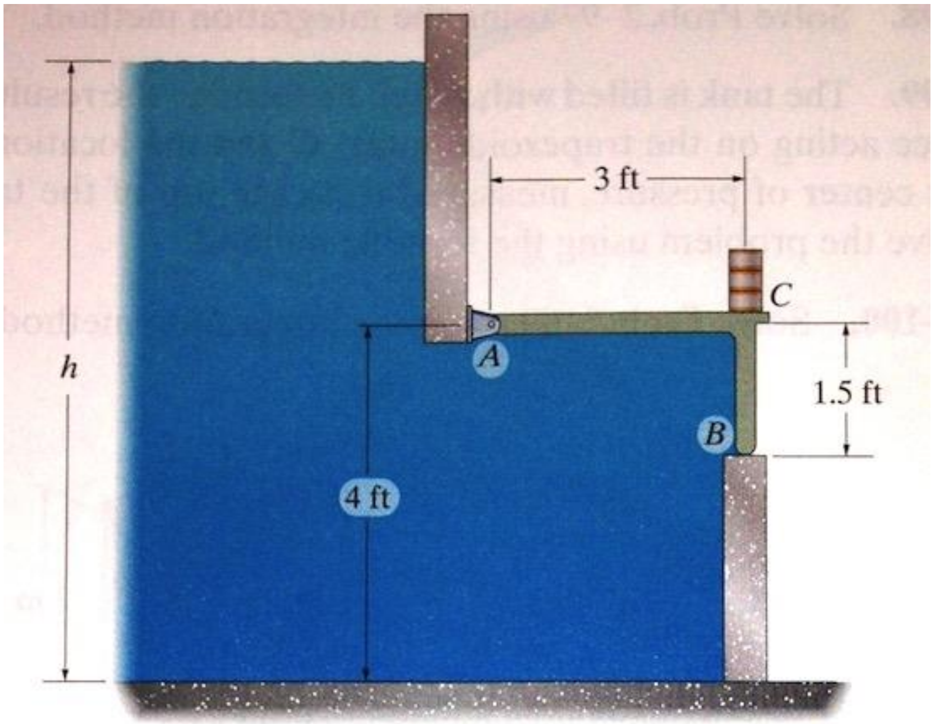
1. The control gate ACB is pinned at *A* and rests on the smooth surface *B*. If the counterweight *C* is 2000 lb, determine the maximum depth of water *h* in the reservoir before the gate begins to open. The gate has a width of 3 ft and has negligible mass compared to the counterweight.



2. The aortic arch can be considered as a curved tube with pulsatile flow. Some dimensional variables are the diameter of the tube, D, the length of the tube L, the radius of curvature, r, the pulsatile frequency, f, the dynamic viscosity of blood, μ, the density of the blood, ρ, mean velocity in the tube, V, and the time of the experiment, T.

a) Derive four independent non-dimensional parameters that would apply to this fluid dynamic situation. Choose the parameters for relevance to the dynamic behavior of the flow. Optional: Do you know the common names of these non-dimensional numbers?

b) It is sometimes useful to model a flow situation in a laboratory with another fluid, such as water. Assuming we keep the dimensions of the model the same as the true aorta, if the kinematic viscosity of water is ¼ the kinematic viscosity of whole blood, what should be the mean velocity of water in the experiment to simulate the fluid dynamics?

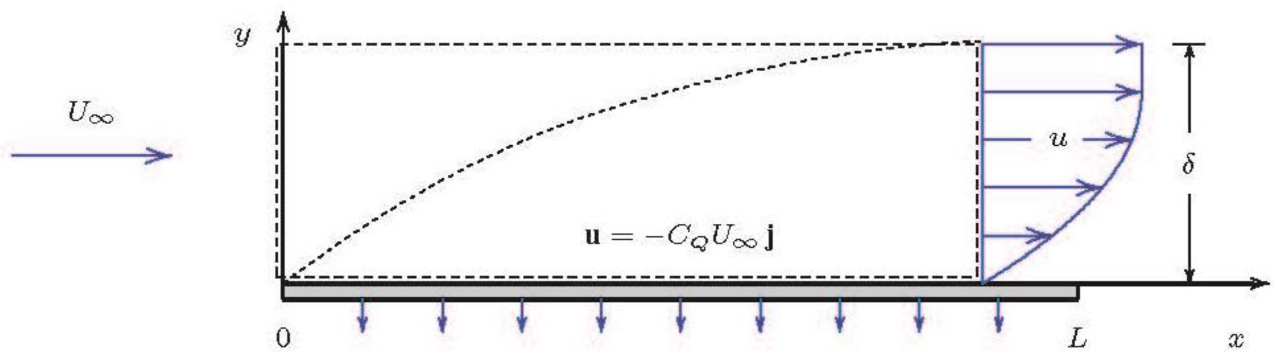
c) It is common for hemodynamicists to use an alternate form of a non-dimensional parameter called the Womersley number for pulsatile flows in tubes: $Wo = R\sqrt{(2\pi f\rho/\mu)}$, where R is the tube radius and $2\pi f$ is the angular frequency. Can you combine some of the parameters you chose in Part **a** to derive the Womersley number? What is the interpretation of force balance for the Womersley number?

d) We would like to simulate the pulsatile nature of blood, but use water instead. What should the frequency of pulsatility be for the water model, using dimensional similarity?

e) If we are to quantify **the shear stress** of the fluid on the aortic wall, is any conversion necessary from the laboratory measurements to the true aorta?

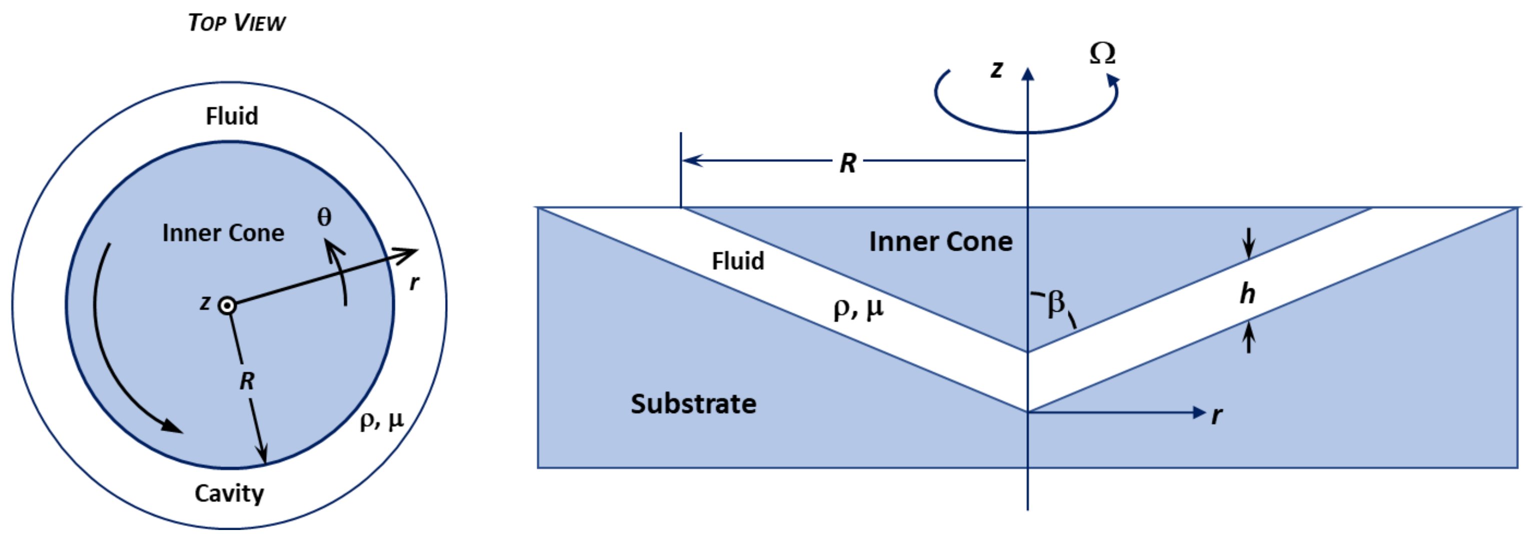
3. Consider steady, incompressible, viscous flow above a flat plate of length $\boldsymbol{L}$ with surface mass removal, which is referred to as suction. The velocity above the plate is given by $u(x,y) = U_\infty f\left(\frac{y}{\delta}\right)$ where U_∞ is the constant freestream velocity and $\delta(x)$ is the thickness of the viscous region. Pressure is constant and equal to p_∞ and you can assume that $u = U_\infty$ for $y \geq \delta$. The surface velocity is given by $\mathbf{u} = -C_Q U_\infty \mathbf{j}$, where C_Q is the constant suction coefficient. Show that, with ρ denoting fluid density and $\dot{m} = \rho U_\infty C_Q L$, the drag force per unit width out of the page is

$$F_D = \dot{m} U_\infty + \int_0^\delta \rho(U_\infty - u)\, u\, dy$$



You can use the stationary rectangular control volume shown in the figure for analysis.

4. A pointing down cone with the outer radius R is located a small distance above a conical cavity in the substrate. The cone and the cavity are concentric and have an equal half-angle of β. The cone rotates at a constant angular speed Ω. The gap between the cone and the cavity has a constant vertical (z) dimension of h, and is completely filled with a Newtonian fluid of constant density ρ and constant viscosity μ. Assume that the flow is steady, unidirectional, laminar, and $h \ll R$.



a) What are the no-slip boundary conditions on the velocity field $\vec{\mathbf{V}}$ for the flow in the gap between the cone and the cavity?

b) List all <u>additional</u> assumptions (beyond those given in the problem statement), and simplify the relevant governing equations. In words, what drives this flow?

c) Determine the velocity field $\vec{\mathbf{V}}$. Note that you are <u>not</u> required to solve the governing equations. <u>Hint</u>: use the boundary condition at the cavity surface to "guess" the form of the velocity field that ensures that this no-slip condition is automatically satisfied.

d) Determine the shear stresses for this flow.

e) What are $\vec{\mathbf{V}}$ and the shear stresses for this flow in the limit where $\beta \rightarrow \pi/2$?

For your reference, the Navier-Stokes equations in cylindrical polar coordinates are:

$$\rho\left(\frac{\partial V_r}{\partial t}+V_r\frac{\partial V_r}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta}-\frac{V_\theta^2}{r}+V_z\frac{\partial V_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\rho g_r+\mu\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial(rV_r)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial^2 V_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial V_\theta}{\partial \theta}+\frac{\partial^2 V_r}{\partial z^2}\right\} \qquad (r)$$

$$\rho\left(\frac{\partial V_\theta}{\partial t}+V_r\frac{\partial V_\theta}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta}+\frac{V_rV_\theta}{r}+V_z\frac{\partial V_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}+\rho g_\theta+\mu\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial(rV_\theta)}{\partial r}\right]+\frac{1}{r^2}\frac{\partial^2 V_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial V_r}{\partial \theta}+\frac{\partial^2 V_\theta}{\partial z^2}\right\} \qquad (\theta)$$

$$\rho\left(\frac{\partial V_z}{\partial t}+V_r\frac{\partial V_z}{\partial r}+\frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta}+V_z\frac{\partial V_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\rho g_z+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2}+\frac{\partial^2 V_z}{\partial z^2}\right] \qquad (z)$$

The shear stresses for a Newtonian fluid in cylindrical polar coordinates are:

$$\tau_{r\theta}=\tau_{\theta r}=\mu\left[r\frac{\partial}{\partial r}\left(\frac{V_\theta}{r}\right)+\frac{1}{r}\frac{\partial V_r}{\partial \theta}\right] \qquad \tau_{\theta z}=\tau_{z\theta}=\mu\left(\frac{\partial V_\theta}{\partial z}+\frac{1}{r}\frac{\partial V_z}{\partial \theta}\right) \qquad \tau_{rz}=\tau_{zr}=\mu\left(\frac{\partial V_r}{\partial z}+\frac{\partial V_z}{\partial r}\right)$$