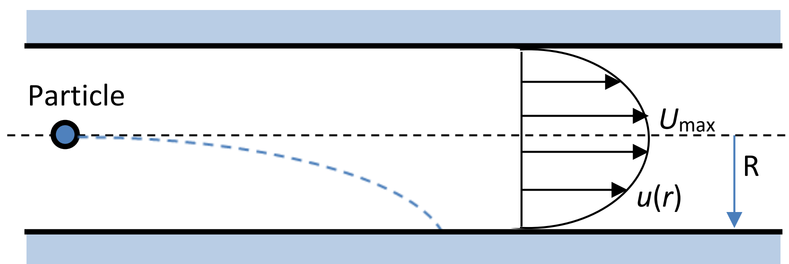
1. A particle-separation system is designed to separate particles based on their density. In this system, spherical particles with density $\rho + \Delta\rho$ and radius, a, are injected at the flow speed on the centerline of a circular tube (radius R) in which the flow of a carrier Newtonian fluid (flow rate Q, centerline velocity U_{max}, density ρ, and viscosity μ) is fully-developed and laminar (Poiseuille flow). Following the injection, each particle begins to gradually descend under gravity until it reaches and then firmly adheres to the tube wall.

It can be assumed that: ***i.*** The flow in the pipe is unperturbed by the presence of the particle; ***ii.*** The pressure distribution in the vertical direction is hydrostatic; ***iii.*** The particle experiences Stokes drag force $F_d = 6\pi\mu a V_r$ where V_r is the particle's vertical velocity relative to fluid; ***iv.*** Following its injection, the particle immediately reaches and maintains its terminal falling; and ***v.*** The particle axial speed instantaneously adjusts to be the same as the fluid's speed, $u(r)$ at the same radial position, r.

Determine the axial length along the tube relative to the injection point that is needed for the particle to reach the tube wall and become separated from the carrier fluid.



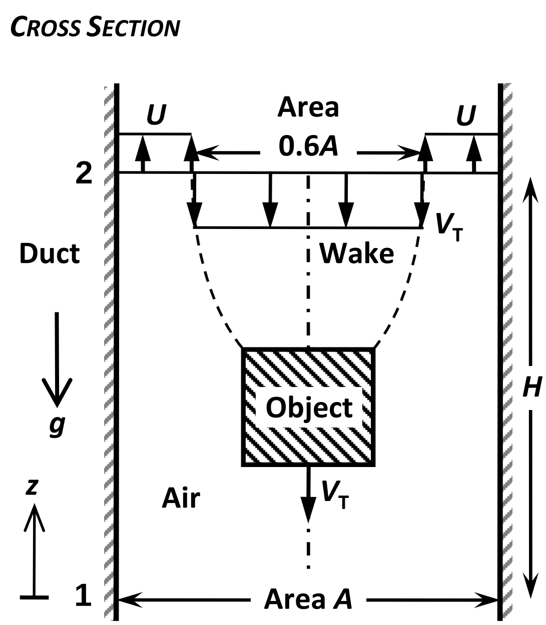
2. In a chemical plant, it is desired to characterize the volume flow rate Q of a prototype axial water pump (viscosity μ_w and density ρ_w) over a range of operating conditions by testing a 1/10 geometrically-scaled model using oil (viscosity μ_o and density ρ_o). The prototype's characteristic diameter is D, its rotation speed is ω, and the desired pressure rise across the pump (in water) is Δp.

a) Using D, ω, and ρ_w as repeating variables, develop a set of dimensionless groups to determine the dependence of Q on the other variables.

b) What is the required rotation speed of the model ω_m for a viable test if the scale model is tested in oil (viscosity μ_o and density ρ_o)?

3. A cylindrical object falls down a long circular duct of cross-sectional area A filled with air of constant density ρ at a constant terminal velocity of V_T. Some distance below the object across Section **1**, the air is at rest at pressure p_1. At Section **2** above the object (a distance H above Section **1**), the air is affected by the falling object. There are two domains: the inner wake with an area of $0.6A$ and uniform speed V_T, and the outer flow with an unknown speed U.

It can be assumed that viscous effects can be neglected between Sections **1** and **2**, the mass of the air between Sections **1** and **2** is negligible compared to the mass of the object, and that the flow is steady.



a) Find the speed of the air U at Section 2 outside the wake.

b) Find the pressure p_2 at Section **2** if the pressure is uniform across Section **2**.

c) Find the mass of the object M.

4. Consider steady, pressure-driven, fully-developed laminar flow of a Newtonian fluid (density ρ, and viscosity μ) in a circular pipe of diameter D (Poiseuille flow).

a) Using the axial (z) Navier-Stokes equation in cylindrical polar coordinates (below) determine the streamwise velocity $V_z(r)$ and stress $\tau_{rz} = \tau_{zr}$ distributions within the pipe.

It may be assumed that the flow velocity is only in the axial direction, the pressure gradient is constant, and gravitational effects are negligible. Please list all additional assumptions that are needed for your derivation.

b) Derive a relationship for the dependence on the Reynolds number of the friction factor, f, that is used in the equation for the head loss, $H_L = (L/D)\cdot(V^2/2g)\cdot f$, where L is the length of the pipe.

The z-component of the Navier-Stokes equations in cylindrical polar coordinates is:

$$\rho\left(\frac{\partial V_z}{\partial t} + V_r\frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta} + V_z\frac{\partial V_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 V_z}{\partial \theta^2} + \frac{\partial^2 V_z}{\partial z^2}\right]$$

and the shear stress is: $\tau_{rz} = \tau_{zr} = \mu\frac{\partial V_z}{\partial r}$.