GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam – Spring Semester 2020

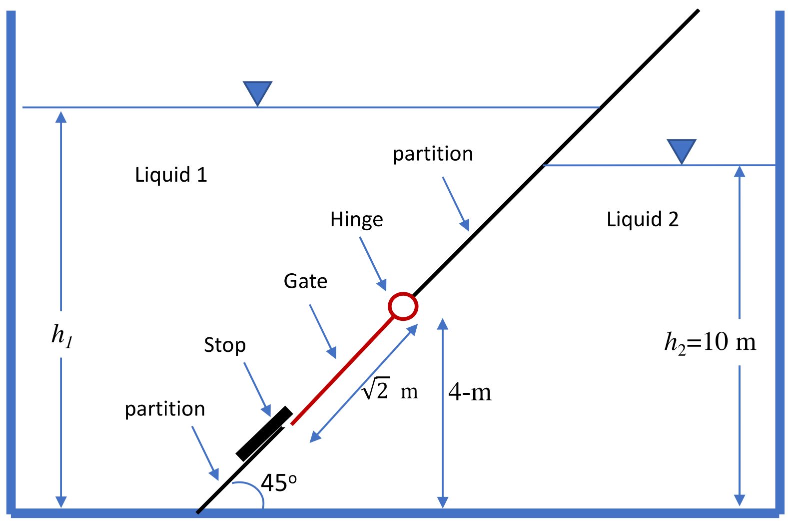
FLUID MECHANICS

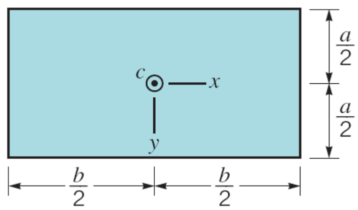
EXAM AREA

Assigned Number **(DO NOT SIGN YOUR NAME)**

✷ Please sign your **name** on the back of this page —

1. Two different liquids (Liquids 1 and 2) are separated by a stationary partition oriented at 45° angle (as shown in the figure). Liquid 2 has a density that is two times larger than the density of liquid 1 ($\rho_2 = 2\rho_1$). A square Gate that is $\sqrt{2}$ (square root of 2) meters long and wide, is attached to the partition with a hinge located 4-m above the bottom. The Gate can only open in the counterclockwise direction because of a Stop at the tip. If h_2=10 m, determine the maximum height of liquid 1, h_1, that the gate can contain and still remain closed.
Assume the hinge is frictionless and neglect the thickness and weight of the Gate.





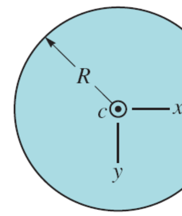
$A = ba$

$I_{xc} = \frac{1}{12} ba^3$

$I_{yc} = \frac{1}{12} ab^3$

$I_{xyc} = 0$

(a) Rectangle

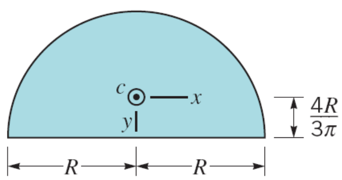


$A = \pi R^2$

$I_{xc} = I_{yc} = \frac{\pi R^4}{4}$

$I_{xyc} = 0$

(b) Circle



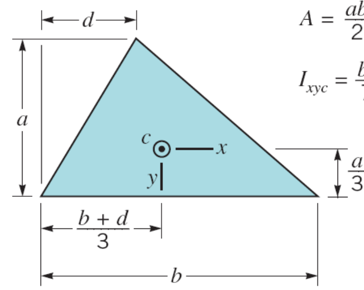
$A = \frac{\pi R^2}{2}$

$I_{xc} = 0.1098 R^4$

$I_{yc} = 0.3927 R^4$

$I_{xyc} = 0$

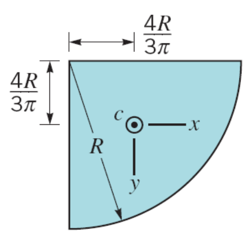
(c) Semicircle



$A = \frac{ab}{2}$ $\qquad I_{xc} = \frac{ba^3}{36}$

$I_{xyc} = \frac{ba^2}{72}(b - 2d)$

(d) Triangle



$A = \frac{\pi R^2}{4}$

$I_{xc} = I_{yc} = 0.05488 R^4$

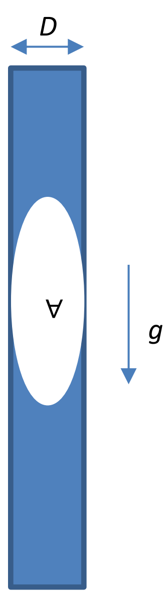
$I_{xyc} = -0.01647 R^4$

(e) Quarter circle

2. In some forms of airway disease, the airways become flooded with liquid, and bubbles can become trapped in these airways. When the bubbles are driven by buoyancy through the airways, shear stresses are generated on the airway wall, and it has been hypothesized that the epithelium covering the airways can be damaged by this shear stress. The problem of analyzing motion of a long bubble through a small airway is complicated. One possible way to address this problem is to build an experimental model, measure the velocity at which the bubble moves through the model system, and then use this result to determine the velocity at which a bubble might pass through a fluid filled airway.

Assume that, in the lung, we wish to explore the motion of a bubble (of volume $\forall = 8 \times 10^6\ \mu m^3$) through a vertical airway of diameter ($D = 100\ \mu m$). The fluid filling this airway has a density (ρ) that is the same as water, a viscosity (μ) that is ten times that of water (8.9×10^{-4} Pa*s), and a surface tension (σ), with respect to air, that is a quarter that of water. The gravitational constant is g in both systems.

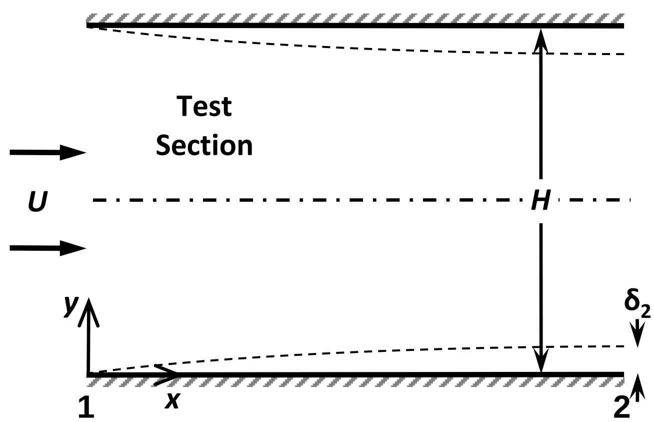
a) We wish to build a model that is dynamically similar to the situation in the lung. An air bubble will be place into a vertical fluid-filled tube of diameter 0.4 mm. Determine the bubble volume and a set of liquid characteristics such that similarity is achieved. Assume the test fluid to be used has a surface tension similar to water.
b) Assume that, for this set of experimental conditions, the bubble velocity in the experimental model is measured to be 1 mm/s. Predict the velocity of the bubble in the lung.
c) In the model experiment, a laser was used to measure the gap between the bubble and the tube wall. The closest approach of the bubble to the wall was a distance of $50\ \mu m$. Using the experimental results, roughly estimate what the peak shear stress on the cells of the lungs is? (In making your estimate, you may assume the bubble behaves like a rigid body.)



3. A small low-speed wind tunnel has a horizontal test section of height H, where the effects of the side walls of the test section are negligible. The flow at the inlet of the test section (Section **1**) is uniform and of speed U. A laminar boundary layer forms on both the upper and lower walls of the test section, and the velocity profile in the boundary layer on the bottom wall at the exit of the test section (Section **2**) has a cubic profile:

$$\frac{u}{U_2} = \frac{3}{2}\frac{y}{\delta_2} - \frac{1}{2}\left(\frac{y}{\delta_2}\right)^3$$

where U_2 is the speed at the outer edge of the boundary layer and δ_2 is the thickness of the boundary layer. The air flowing through the test section has a constant density ρ and constant viscosity μ.



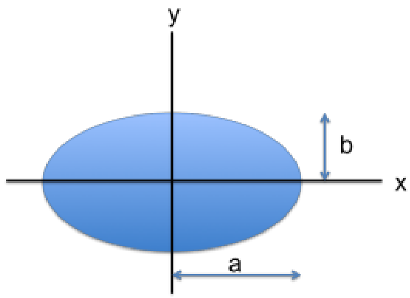
Please define your control volume and list all additional assumptions.

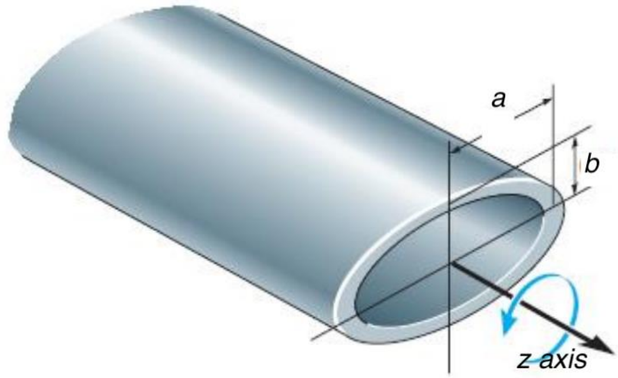
a) Find the speed U_2.

b) Estimate the pressure drop across the test section Δp if the pressure is uniform across the inlet and exit.

c) Estimate the x-component of the total force $\vec{\mathbf{F}}_x$ exerted by the flow on the top and bottom walls of the test section.

4.





A. For flow in an elliptical pipe with major axis of radius *a* and minor axis of radius *b*, state the boundary conditions and assumptions needed to derive an equation for the velocity as a function of position.

B. Using cartesian coordinates, what is the differential equation for conservation of mass for the x, y, and z directions? What are the *u* and *v* velocities?

C. Write the Navier-Stokes equations with the relevant terms for this flow.

D. If pressure, p, varies linearly along z, ($p = -Gz$) what is the differential equation for velocity, *w*, in the z direction?

E. The form of the equation for velocity *w* is $w = Ax^2 + By^2 + C$. The equation for an ellipse is $x^2/a^2 + y^2/b^2 = 1$ for a major width of 2a and minor height of 2b. How would you solve for A, B, and C?

F. For the elliptical pipe with major axis a and minor axis b, where will shear stress be greatest and where will it be smallest?

G. Will this differ for flows at low or high Reynolds number and why?