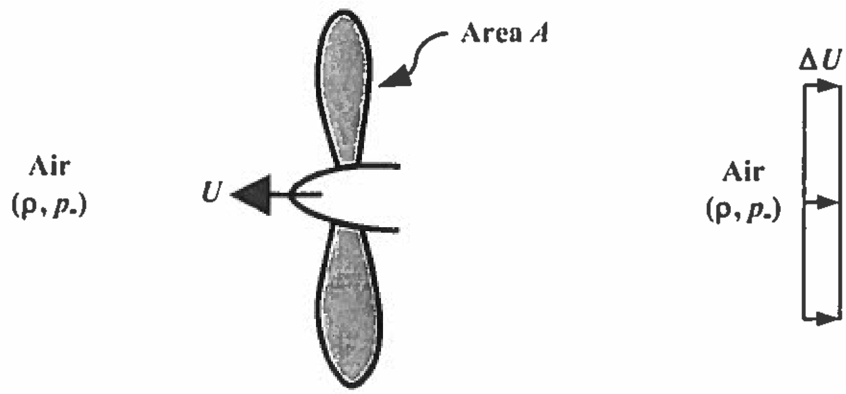
1. The propeller on an airplane traveling steadily at a constant speed of U through air of constant density ρ and pressure p_∞ has a projected area A. The air is at rest some distance upstream of the propeller, and is accelerated by the propeller. The flow in the wake of the propeller, which will be smaller than the propeller, can be modeled some distance downstream of the propeller as uniform flow of constant speed ΔU.



To analyze this flow in a reference frame <u>fixed on the propeller</u>, divide the flow into three parts: A) the flow upstream of the propeller starting where the air is at rest and ending just upstream of the propeller; B) the flow across just the propeller; and C) the flow downstream of the propeller starting just downstream of the propeller and ending at the uniform wake flow. Then assume that there is no loss in parts A and C, and that the velocity is uniform and constant across part B.

a) Sketch control volumes and velocity distributions at the control surfaces for the flow in parts A, B, and C.

b) Determine the pressure change across the propeller Δp, and explain whether the pressure increases or decreases across the propeller.

c) Finally, determine the magnitude of the thrust T developed by the propeller, *i.e.*, the horizontal force component due to the propeller on the flow.

2. Consider two long horizontal concentric inner and outer tubes having radii R_i (external) and R_o (internal), respectively, as shown schematically in the Figures 1a and b below (tube lengths >> R_i and R_o). The annular space between the tubes is filled with Newtonian fluid (viscosity μ and density ρ). It is desired to set the fluid within the annular space in motion in the absence of an external pump by moving either the outer or the inner tube along its axis with constant velocity U, while the other tube is stationary. Using appropriate analysis, determine the velocity distribution within the annular domain and the force that is required to move the tube for each translation mode (i.e., using the outer or inner tubes in Figures 1a and b). Which translation mode requires less power? ***Justify and explain your answer.***

It may be assumed that: **i.** the transients associated with the onset of the motion have died out, **ii.** the motion of the fluid becomes fully-developed and laminar, **iii.** the mass of each tube is negligible, and **iv.** the motion of each tube is frictionless.

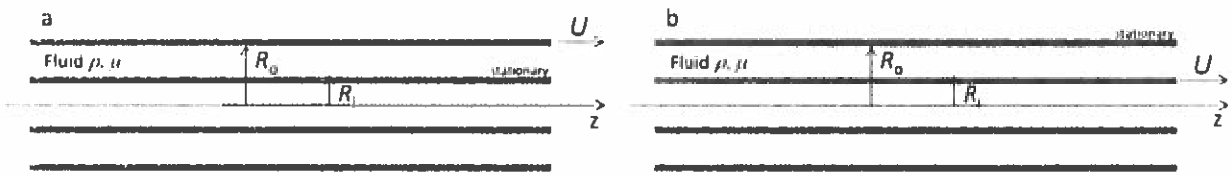


Figure 1

Equation of Continuity

$$\frac{\partial \rho}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta)+\frac{\partial}{\partial z}(\rho v_z)=0$$

Navier Stokes Equations

$$\rho\left(\frac{\partial v_r}{\partial t}+v_r\frac{\partial v_r}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}-\frac{v_\theta^2}{r}+v_z\frac{\partial v_r}{\partial z}\right)=-\frac{\partial p}{\partial r}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right)+\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}-\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}+\frac{\partial^2 v_r}{\partial z^2}\right]+\rho g_r$$

$$\rho\left(\frac{\partial v_\theta}{\partial t}+v_r\frac{\partial v_\theta}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}+\frac{v_r v_\theta}{r}+v_z\frac{\partial v_\theta}{\partial z}\right)=-\frac{1}{r}\frac{\partial p}{\partial \theta}$$
$$+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)\right)+\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}+\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}+\frac{\partial^2 v_\theta}{\partial z^2}\right]+\rho g_\theta$$

$$\rho\left(\frac{\partial v_z}{\partial t}+v_r\frac{\partial v_z}{\partial r}+\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}+v_z\frac{\partial v_z}{\partial z}\right)=-\frac{\partial p}{\partial z}+\mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}+\frac{\partial^2 v_z}{\partial z^2}\right]+\rho g_z$$

3. The pressure drop per unit length that would occur along a blood vessel segment can be modeled as a function of the diameter of the vessel, time, the relevant fluid properties of the blood, the heart rate, and the average velocity of the blood. You propose to design an experiment using a glass tube that is 5 times larger than the vessel you wish to model. You will use a fluid that has a density similar to blood, but a viscosity that is 50% less. What are your similarity requirements? How does the pressure drop per length that you measure in the tube compare to what you would expect to measure in the actual vessel?

Note: Blood flow can be turbulent and roughness features on a blood vessel are not important.

A cylindrical container with radius R is filled with two immiscible fluids with densities ρ_1 and ρ_2, respectively, as shown in Fig. A. The container has a flat bottom wall and vertical side walls and spin with a constant angular velocity ω. Assume that $\rho_1 < \rho_2$, the fluids have identical volumes $V_1 = V_2 = V$, and that surface tension effects can be neglected.

1. Find the magnitude of the critical angular velocity ω_{cr} at which the container bottom will be exposed.
2. Find expressions describing the positions of the free surface and the interface between the two fluids when $\omega = \omega_{cr}$. Make a sketch showing the free surface and the interface between the fluids in this situation.
3. For $\omega = \omega_{cr}$, sketch pressure distribution along the side wall and find the tension force in the side wall due to the rotating fluid (see Fig. B).

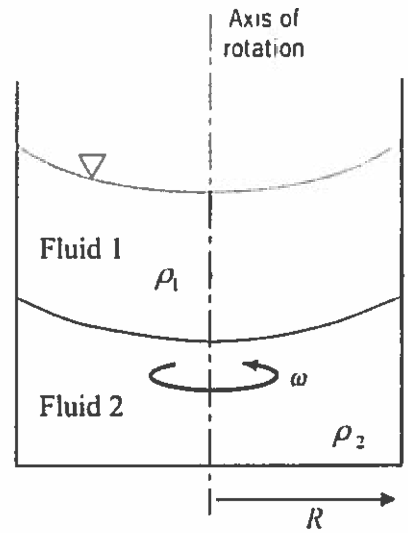


Fig. A

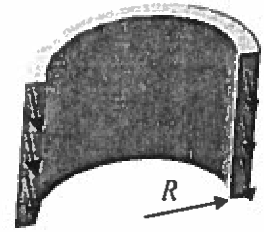


Fig. B