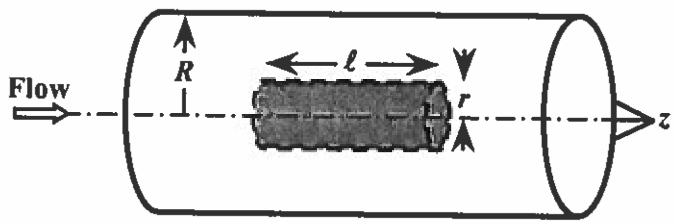
1. Consider the fully-developed, steady, laminar and unidirectional flow of a power-law fluid of constant density ρ through a horizontal pipe of inner radius R driven by a known axial pressure gradient $\Delta p / L$ (note that $\Delta p / L > 0$), which you may assume is constant. The velocity field in this flow is $\vec{V} = V_z(r)\hat{\mathbf{z}}$, and the viscous stress for a power-law fluid in such a flow is $\tau_{rz} = K(dV_z / dr)^N$.

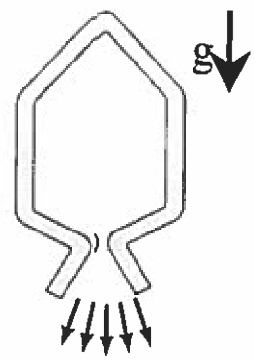


a) Sketch a free-body diagram of the forces acting along the z-direction on the shaded fluid element shown above, which is a cylinder of fluid of radius r and axial dimension l centered about the pipe centerline. Clearly identify all of your forces in words.

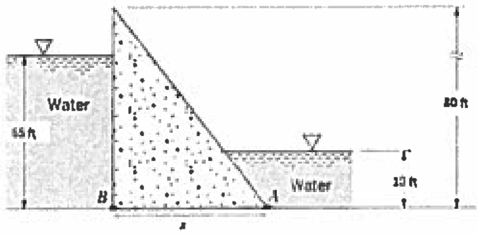
b) If this flow has zero acceleration, determine the velocity in this flow $\vec{V}$. As a check of your answer, show that your solution reduces to the parabolic profile for a Newtonian fluid, which is the special case of a power-law fluid where $K = \mu$ and $N = 1$.

c) Finally, determine the volume flow rate Q across a cross-section of the pipe.

2. A rocket is fired vertically from the surface of Moon. The initial rocket mass with the fuel is m_0. The pressure, density, and velocity of the exhaust gas are P_e, ρ_e, and V_e at the nozzle exit that has a cross-sectional area A_e. The gravitational acceleration is $g = const$. Find how the velocity of the rocket changes after the launch.



3. The thrust due to any one of a family of geometrically similar airplane propellers is to be determined experimentally from a wind tunnel test with a model. By means of dimensional analysis, find suitable nondimensional parameters for plotting the test results. The thrust F_T depends on speed of rotation ω, speed of advance V_0, diameter D, air viscosity μ, density ρ, and speed of sound c.



4. Find the length of the base of the dam, x, necessary to keep the dam from toppling. Assume that water leaks very slowly under the foundation between the two reservoirs and that the specific weight of the concrete dam is 150 lb/ft^3.