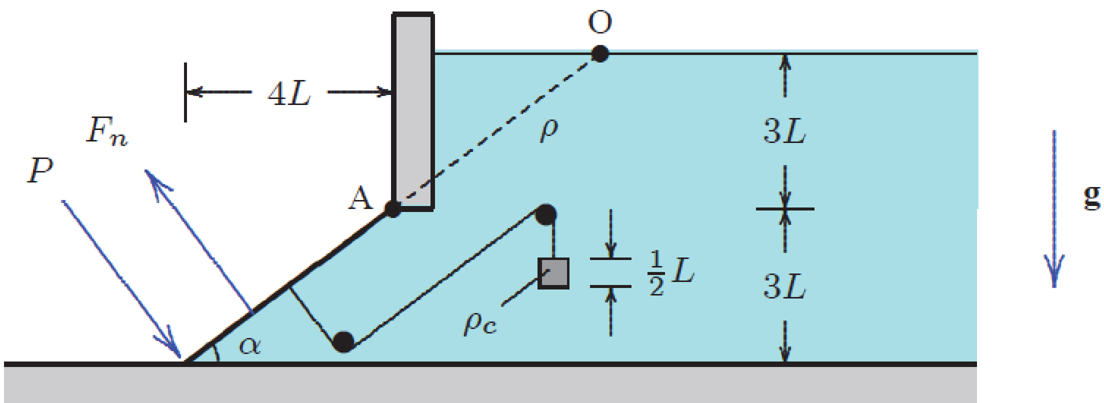
1.



A rectangular gate of width 6L out of the page is hinged at point A and holds back a fluid of density ρ as shown. A cube of density $\rho_c = 5\rho$ and side ½ L is connected to the gate through its centroid with a pulley arrangement so that the weight exerts a force normal to the gate. Neglecting the weight of the gate, determine the minimum force, P, required to hold the gate closed in terms of ρ, L and gravitational acceleration, g.

2. Consider an infinite flat plate at the bottom of an infinitely deep pool of liquid where the plate has an in-plane oscillation. Let the plate lie in the *z*-plane of the Cartesian coordinate system, so oriented that the oscillation is along the x-axis. Due to no-slip condition, the velocity of the liquid at the plate is given by

$u = A\cos(\omega t)$ at $z=0$ and $u = 0$ at $z \to \infty$

The Navier-Stokes equations in Cartesian coordinates are

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right)=-\frac{\partial p}{\partial x}+\rho g_x+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right)=-\frac{\partial p}{\partial y}+\rho g_y+\mu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$\rho\left(\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right)=-\frac{\partial p}{\partial z}+\rho g_z+\mu\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$

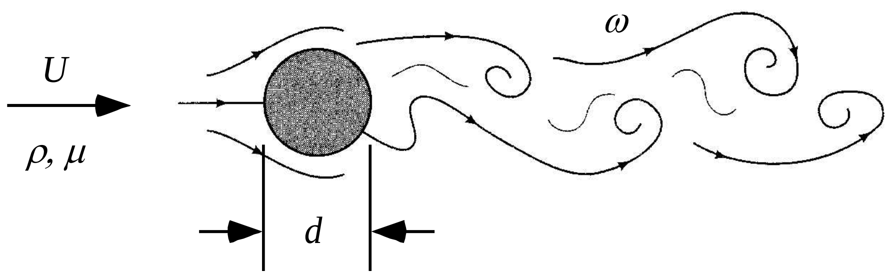
(a) Find the velocity field $u(z,t)$, assuming the solution is in the form

$u = f(z)\cos(\omega t) + g(z)\sin(\omega t)$, $v=w=0$, $p=constant$, and velocity decays exponential with *z*.

(b) Explain the reason for the phase-lag in the oscillation of the fluid particles above the plate.

(c) Determine the distance between the fluid particles that oscillate in-phase.

3.



We know that, for some range of flow speeds, a periodic "Kármán vortex street" of shed vortices appears behind a cylinder, as shown above.

(a) One suspects that there is a relationship between the frequency ω of the shed vortices, the velocity U, diameter d, density ρ, and viscosity μ. Using dimensional analysis, determine how many dimensionless groups should govern this problem and calculate them specifically. **Do NOT form groups "by inspection." Work out the details.**

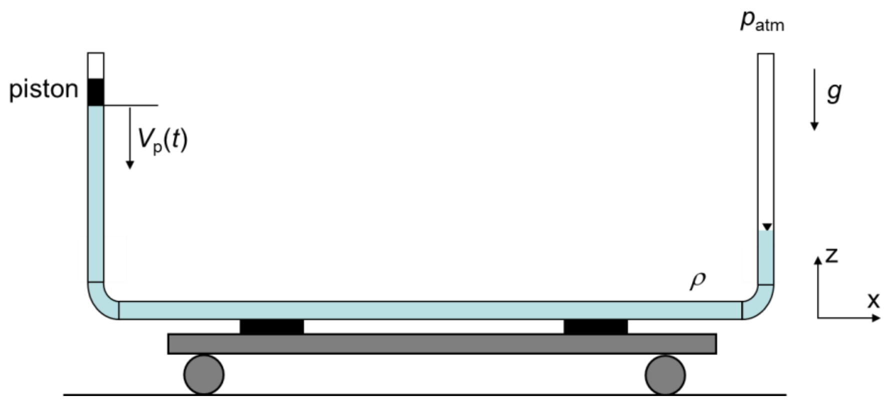
(b) The x-component of the Navier-Stokes equations for 2-D flow in Cartesian coordinates is (neglecting body force)

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

Scale this equation using ω^{-1} as the time scale, the obvious length and velocity scales and assuming that the pressure scale balances inertia. Show that the equation can be manipulated to contain the dimensionless parameters identified in part a. Would you typically work with this particular equation for this flow? Why or why not?

(c) This phenomenon is observed to occur behind a cylinder of diameter 5 mm in air with $\nu = 1.51\times10^{-5}\ \text{m}^2/s$ at a speed of $U = 30\ \text{cm/s}$. If the same cylinder is placed into a water with $\nu = 1.00\times10^{-6}\ \text{m}^2/s$, at what speed should one observe the phenomenon? What is the relationship between the frequencies ω_a and ω_w observed in air and water?

4.



A tube of cross sectional area A is attached to a cart that rests on a frictionless track along the x axis as shown in the figure. The tube is filled with liquid of density ρ. It is fitted with an electromagnetically operated frictionless piston on the left, and it is open to the atmosphere on the right. Initially, the cart is at rest, and its position relative to the fixed frame of reference x-z is X_o. At time $t = 0$ the piston is set in motion so that it moves downward with speed $V_p = K \cdot t$ where K is a constant.
It may be assumed that: ***i.*** The piston is set into motion smoothly, and instantly and any initial transients are negligible; ***ii.*** The motion of the cart is frictionless; ***iii.*** The masses of the cart, the tube, and the piston are negligible compared to the mass of the liquid; ***iv.*** The velocity distributions of the fluid through all cross sections of the straight segments of the tube are uniform, and corner effects and losses in the 90° bends are negligible (the characteristic scale of the bends is much smaller than the length of each of the straight segments of the pipe); and ***v.*** The length of the horizontal segment of the tube is L and of each of the vertical segments $L/2$.

(a) Using an appropriate form of the momentum equation, determine whether the position of the cart changes with time after the piston is set in motion, and, if so, determine how.

(b) How, if at all, does the position of the cart change if at $t = t_s$ after the motion of the piston begins, the speed of the piston becomes invariant with time $V_p = V_s$.