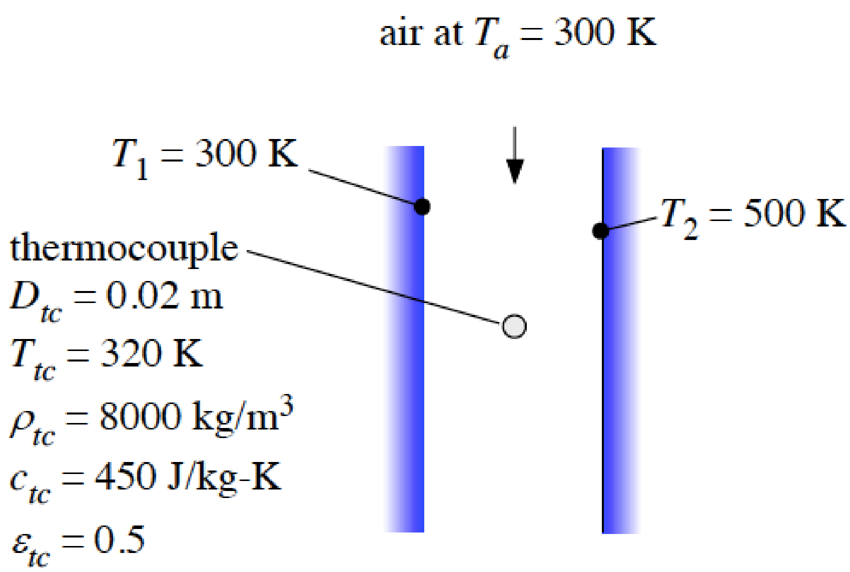
Heat Transfer

1. A spherical thermocouple of diameter D_{tc}=0.02m is made of a material with thermal conductivity k_{tc} = 30 W/mK, density ρ_{tc} = 8000 kg/m^3, and specific heat c_{tc} =450 J/kgK. The thermocouple is at a uniform temperature T_{tc} = 320K and is inserted between two infinitely long parallel walls to measure the temperature of a flowing air stream. The emissivity of the thermocouple's surface is ε_{tc} =0.5 while both plates have an emissivity of 1. The first plate has a temperature T_1 = 300K and the other plate has a temperature of T_2 = 500K. The air flowing between the plates is at a velocity of 3.5 m/s and temperature of 300K. The Nu correlation for a sphere is given by:

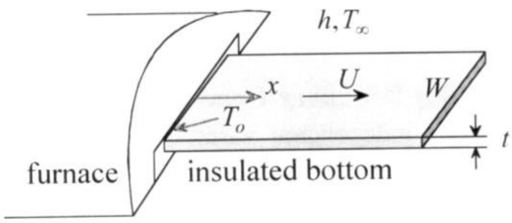
$$\mathrm{Nu}_{sph} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}]\,\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$



Find:

1) The net rate of heat transfer to the thermocouple.
2) Determine the initial rate of change of the temperature of the thermocouple.
3) To improve the accuracy of the measurement, it is suggested that a thin coating be put on the thermocouple that provides negligible thermal resistance, but changes the emissivity. Coatings exist that can increase or decrease the emissivity. Which one should you choose to improve the accuracy of the temperature measurement?

2. A thin metal foil of thickness t and width W is heated in a furnace to temperature T_o to grow carbon nanotubes on its surface. The foil moves on a conveyor belt traveling with velocity U. It is cooled by convection outside the furnace by an ambient fluid at T_∞. The heat transfer coefficient is h. Assume steady state, $Bi < 0.1$, negligible radiation and no heat transfer from the foil to the conveyor belt.



(a) Derive the differential equation and give the boundary conditions required to determine an expression for the temperature distribution in the foil (Do NOT solve the differential equation). Clearly state any assumptions made.

(b) Sketch $\frac{T(x)-T_\infty}{T_0-T_\infty}$ versus position x along the foil for $U \approx 0$, $U \to \infty$, and two intermediate velocities.

(c) Re-sketch one of the intermediate curves from part (b) on a new plot and show with a new curve on the same plot how the plot would change if the thickness, t, is increased (with Bi still less than 0.1). Velocity U remains constant in both cases. Give the rationale for your answer as well.

3. Consider the high speed laminar, steady, two-dimensional boundary layer flow of a gas over an adiabatic surface. The free stream velocity and temperature are U_∞ and T_∞ respectively.

1) Assuming viscous dissipation to be important write the boundary layer form of the equations. Clearly state the approximations made in writing this form of the equations from the original governing equations. Also write the boundary conditions. **(25%)**
2) Transform the boundary layer energy equation using the following similarity variables for the distance η and stream function ψ respectively:

$$\eta = \frac{y}{2}\sqrt{\frac{U_\infty}{\nu x}}$$

$$\psi = \nu f(\eta)\sqrt{Re_x}$$

Where x is the distance along the plate starting from the leading edge, y the distance normal to the plate, ν the kinematic viscosity, and Re_x the local Reynolds number. (**35%**)

3) Define a suitable normalized temperature and re-write the energy equation with this normalized temperature. Then show an expected form of the resulting temperature profile across the boundary layer based on the resulting energy equation and the boundary conditions. Where does the maximum temperature in the boundary layer occur ? (**40%**)

Note: The applicable form of the two-dimensional energy equation with viscous dissipation is provided below:

$$\rho c_p \left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \mu\Phi + \dot{q}$$

Where: $\Phi \equiv \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right] - \frac{2}{3}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right)^2$