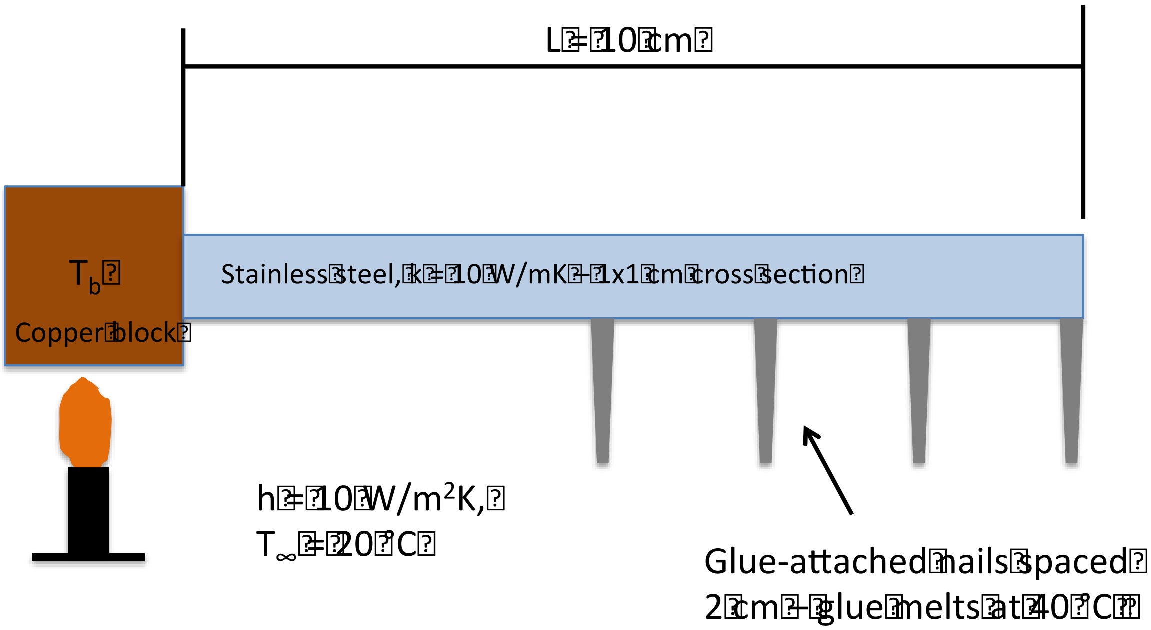
Ph.D. Qualifying Examination, Fall 2013

Heat Transfer

1.



You designed an experiment with simple supplies to convey important heat transfer concepts to high school students. A long, square cross section steal rod is welded to a copper block that is heated with a flame to a constant temperature, T_b = 100 °C. Four nails are glued (melting temp of glue is 40 °C) to the rod spaced by 2 cm from the end of the rod. The ambient conditions are fixed as given – radiation can be neglected.

A) Neglecting contact resistance between the rod and heated base, determine which nails will fall from the rod at steady state.

B) What minimum temperature does the copper block need to reach to ensure all nails fall from the rod at steady state?

C) After reaching the temperature in part B, the flame is turned off and the system is allowed to cool. Assuming that the majority of heat is transferred from the copper to the ambient through the steal rod, derive an expression to estimate how long it would take for the maximum temperature in the system to reach a safe to touch temperature of 30 °C.

D) How would this expression change if significant contact resistance existed between the rod and the copper block? How would the required cooling time change?

E) Would any of your assumptions change if the block and rod were both made of copper?

2. Consider the steady flow of a highly viscous oil of dynamic viscosity μ, and thermal conductivity k, through a long, thin metal pipeline of radius R at given volume flow rate of Q. The tube is covered with an insulation of thickness t_{ins} and thermal conductivity k_{ins}, which is exposed to ambient air at a temperature of T_{amb}, and a convective heat transfer coefficient h. Assume the flow is fully developed hydrodynamically at the inlet of the pipe, and enters with a uniform temperature T_{amb}. Neglect axial heat conduction in the fluid.

i. Qualitatively plot the centerline, and pipe surface temperatures along the length of the pipe. Also, plot the radial temperature distributions at the inlet, and two axial locations. Explain the reasoning for the plots. **(20%)**

ii. Write down the governing equations and boundary conditions for determining the radial temperature distribution, far from the inlet where the fluid temperature does not change axially. Clearly explain why certain terms are kept and others dropped. **(15%)**

iii. Determine the radial fluid temperature distribution within the pipe, and the pipe wall temperature. **(65%)**

Note:

1. The laminar fully developed flow profile in a pipe is given by:

u = $-\frac{1}{4\mu}\frac{dp}{dx}(R^2 - r^2)$

2. The thermal energy equation in the radial coordinates is given by:

$$\rho c\left[\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial r}\right] = k\left[\frac{\partial^2 T}{\partial x^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)\right] + \dot{q}$$

where:

$$\dot{q} = \mu\left\{\left(\frac{\partial u}{\partial r} + \frac{\partial v}{\partial x}\right)^2 + 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial r}\right)^2 + \left(\frac{v}{r}\right)^2\right]\right\}$$

3. A small optical window is used to allow light to pass into a large cryostat that will hold a sample in a vacuum environment that must be irradiated during a test. The cryostat is cooled to 77K by liquid nitrogen. The small window is insulated from the cryostat and separates the cryostat environment from a large room which is at 300K. A solar simulator which approximates the irradiation from the sun will be used to irradiate the sample in the cryostat. Before putting the sample into the cryostat and running the experiment, it is desired to determine the temperature of the window separating the cryostat environment from the room. The following properties are known about the window:

   A) The window is thin enough that the temperature gradient across the window is negligible.

   B) The outer surface of the window is exposed to convective heat transfer with a coefficient of h.

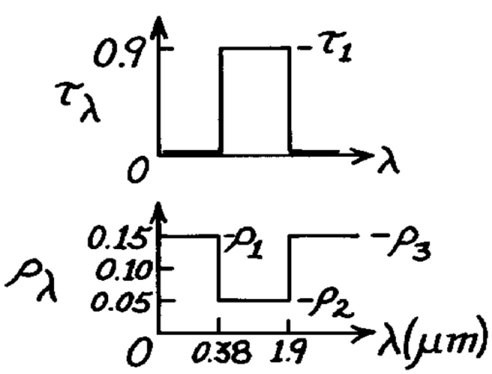
   C) The spectral transmissivity and reflectivity of the window are given below. These values were measured normal to the window surface.

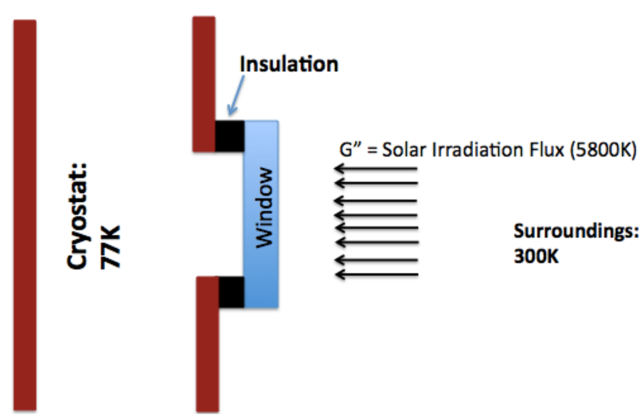
   D) The temperature of the window under steady-state conditions does not increase more than 40°C above room temperature.

It is desired to find the emissivity (ε) and absorptivity (α) of the window. Describe what information and assumptions are needed to find these values. Calculate these values for both the inside and outside surface of the window.

Write an expression that can be used to solve for the temperature of the window.

Write an expression that can be used to determine the net amount of thermal energy transferred into the cryostat. What happens if conduction across the window of thickness a and conductivity k must also be considered?





λT (μm · K)	$F_{(0 \to \lambda)}$	$I_{\lambda,b}(\lambda, T)/\sigma T^5$ (μm · K · sr)$^{-1}$	$\frac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.183120	0.701292	0.970891
2,800	0.227897	0.720239	0.997123
2,898	0.250108	0.722318×10^{-4}	1.000000
3,000	0.273232	0.720254×10^{-4}	0.997143
3,200	0.318102	0.705974	0.977373
3,400	0.361735	0.681544	0.943551
3,600	0.403607	0.650396	0.900429
3,800	0.443382	0.615225×10^{-4}	0.851737
4,000	0.480877	0.578064	0.800291
4,200	0.516014	0.540394	0.748139
4,400	0.548796	0.503253	0.696720
4,600	0.579280	0.467343	0.647004
4,800	0.607559	0.433109	0.599610
5,000	0.633747	0.400813	0.554898
5,200	0.658970	0.370580×10^{-4}	0.513043
5,400	0.680360	0.342445	0.474092
5,600	0.701046	0.316376	0.438002
5,800	0.720158	0.292301	0.404671
6,000	0.737818	0.270121	0.373965
6,200	0.754140	0.249723×10^{-4}	0.345724
6,400	0.769234	0.230985	0.319783
6,600	0.783199	0.213786	0.295973
6,800	0.796129	0.198008	0.274128
7,000	0.808109	0.183534	0.254090
7,200	0.819217	0.170256×10^{-4}	0.235708
7,400	0.829527	0.158073	0.218842
7,600	0.839102	0.146891	0.203360
7,800	0.848005	0.136621	0.189143
8,000	0.856288	0.127185	0.176079
8,500	0.874608	0.106772×10^{-4}	0.147819
9,000	0.890029	0.901463×10^{-5}	0.124801

Table 12.1 *Continued*

λT (μm · K)	$F_{(0 \to \lambda)}$	$I_{\lambda,b}(\lambda, T)/\sigma T^5$ $(\mu\text{m} \cdot \text{K} \cdot \text{sr})^{-1}$	$\dfrac{I_{\lambda,b}(\lambda, T)}{I_{\lambda,b}(\lambda_{max}, T)}$
9,500	0.903085	0.765338	0.105956
10,000	0.914199	0.653279×10^{-5}	0.090442
10,500	0.923710	0.560522	0.077600
11,000	0.931890	0.483321	0.066913
11,500	0.939959	0.418725	0.057970
12,000	0.945098	0.364394×10^{-5}	0.050448
13,000	0.955139	0.279457	0.038689
14,000	0.962898	0.217641	0.030131
15,000	0.969981	0.171866×10^{-5}	0.023794
16,000	0.973814	0.137429	0.019026
18,000	0.980860	0.908240×10^{-6}	0.012574
20,000	0.985602	0.623310	0.008629
25,000	0.992215	0.276474	0.003828
30,000	0.995340	0.140469×10^{-6}	0.001945
40,000	0.997967	0.473891×10^{-7}	0.000656
50,000	0.998953	0.201605	0.000279
75,000	0.999713	0.418597×10^{-8}	0.000058
100,000	0.999905	0.135752	0.000019