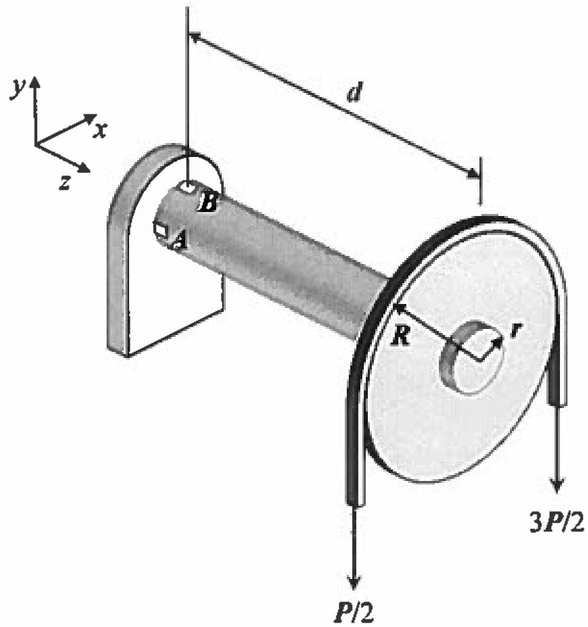
PLEASE NOTE: Answer 3 out of the 4 problems. In case you answer the 4 problems, clearly state which 3 problems you want to be graded.

Problem #1



The shaft shown in the figure above is driven by a belt. The tension in the belt on the tight side is $3P/2$ and the tension on the slack side is $P/2$.

(1) Determine the stress state at point A (located on the lateral side of the shaft).

(2) Determine the stress state at point B (located on the top of the shaft).

(3) If the yield strength of the shaft material is σ_Y, determine the maximum value of P to avoid yielding of the shaft using the von Mises yield criterion.

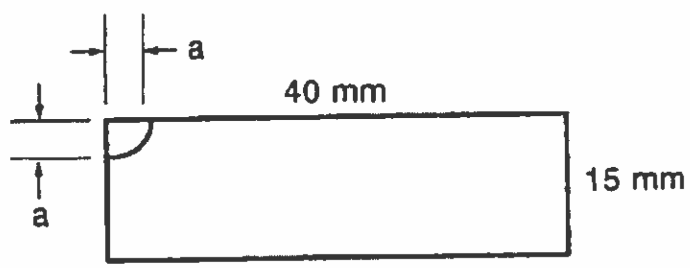
Problem #2

An aircraft structural member made of 7075-T6 aluminum has a cross section as shown below. A quarter circular crack of size $a_i = 0.5$ mm is present, and the member is subjected to a uniaxial stress, S, normal to the cross section. Assume the crack keeps its quarter circular shape as it grows.

(a) Estimate the final crack size, a_f. Assume the crack size responsible for fully plastic yielding is larger than the critical crack size for brittle fracture.

(b) How many cycles between $S_{max} = 336$ and $S_{min} = -68$ MPa can be applied before failure is expected?

(c) How many cycles between $S_{max} = 336$ and $S_{min} = 68$ MPa can be applied before failure is expected?



Material	Yield σ_o MPa (ksi)	Toughness K_{Ic} MPa $\sqrt{m}$ (ksi$\sqrt{in}$)	Walker Equation C_0 mm/cycle (MPa$\sqrt{m}$)^m	C_0 in/cycle (ksi$\sqrt{in}$)^m	m	γ ($R \geq 0$)	γ ($R < 0$)
7075-T6 Al[2]	523 (75.9)	29 (26)	2.71×10^{-8}	1.51×10^{-9}	3.70	0.641	0

Note: The following Walker equation should be used to predict the crack growth rates under various load ratios, $R = S_{min}/S_{max}$:

$$\frac{da}{dN} = C_0(1-R)^{m(\gamma-1)}(\Delta K)^m$$

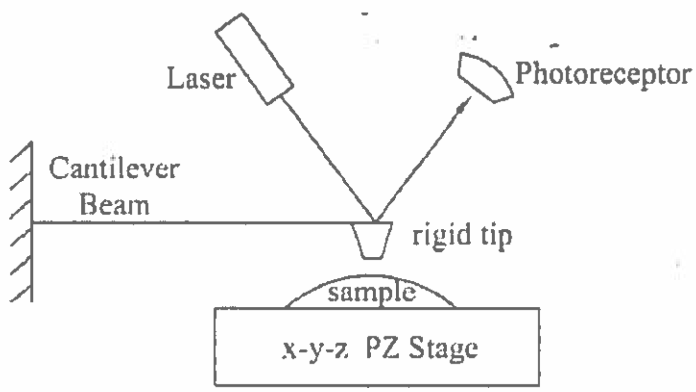
with m, γ and C_0 given in the above table, and with ΔK the *nominal* stress intensity factor range ($\Delta K = K_{max} - K_{min}$).

The stress intensity factor K for that particular crack configuration can be approximated as:

$$K = 0.722 S\sqrt{\pi a}$$

Problem #3

The atomic force microscope (AFM), first reported in 1986, has become a widely used tool to study forces between and within individual molecules on the order of picoNewtons (10^{-12} N) and the topology of surfaces, including cells, with resolution on the order of nanometers. A schematic of the basic design of an AFM probe is shown below. It consists of a cantilever beam with a rigid end tip; the laser and photo-detector are used to measure changes in the angle of the laser light (i.e., the end slope $\phi = \frac{d\nu}{dx}(x = L)$ that are associated with the deflection (ν) of the cantilever. If the cantilever beam (which has a length L, a second moment of area I_{zz}, and a Young's modulus E) is lowered onto the soft sample, the sample will impose a force P on the cantilever tip. Given that we can measure the end slope ϕ of the cantilever, please answer the following.



(a) Let $\delta = \nu(x = L)$ be the deflection at the end of the beam. What is the deflection δ in terms of the material parameters, geometry, and measured values? Note: this result should NOT include the unknown applied load P.

(b) What is the force P in terms of material parameters, geometry, and measured values?

(c) The effective stiffness, k, of the cantilever beam is defined through the relation $P = k\delta$. What is the value of k in terms of material parameters and geometry? Note, given the similarity between $P = k\delta$ and the classical force displacement relation for a spring, $f = \hat{k}\hat{\delta}$, k is often called the AFM spring constant.

(d) If L=400 μm and the beam is rectangular and made of silicon (E=166 GPa) and if the width of the beam $b = 5h$, where h is the height of the beam, what value of h will yield an effective stiffness of k=1.0 N/m?

Beam Equations

$$\sigma_{xx}(x,y) = \frac{-M(x)y}{I_{zz}} \qquad \sigma_{xy} = \frac{V(x)Q_p(y)}{I_{zz}b}$$

$$\frac{dV}{dx} = -q(x) \qquad \frac{dM}{dx} = V(x) \qquad EI_{zz}\frac{d^2v}{dx^2} = M(x)$$

First Moment of Area of a rectangular cross section (above point p)

$$Q_p = \int y\,dA_p = \int_{-b/2}^{b/2}\int_{y}^{h/2} y\,dy\,dz = \frac{b}{2}\left(\frac{h^2}{4} - y\right)$$

Second Moment of Area of a rectangular cross section

$$I_{zz} = \frac{bh^3}{12}$$

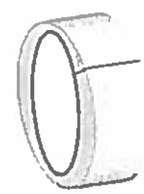
Problem #4

A researcher deposits via sputter deposition at 50°C a thin film of aluminum (200 nm thickness) on a silicon substrate that is 0.5 mm thick (rectangular cross-section: L = 38 mm and W = 10 mm). The substrate and thin film are stress free at the deposition temperature. Assume that there is no delamination possible between the film and the substrate.

	Al	Si	PMMA
E (GPa)	70	173	3.4
ν	0.33	0.1	0.35
α (10^{-6}/°C)	23	3	54000

a) Calculate the change in curvature of the assembled system when the silicon with the aluminum film are removed from the chamber and are at room temperature (25°C).

b) Estimate the thickness of the aluminum thin film that is to be deposited on a polymeric substrate (e.g. PMMA) at the deposition temperature of 50°C (assume again stress free film and substrate at the deposition temperature) if the entire assembly is to become a circular tube (see sketch) at room temperature (25°C).



c) Calculate the stresses that will develop in the aluminum film (part b).