GEORGIA INSTITUTE OF TECHNOLOGY

The George W. Woodruff
School of Mechanical Engineering

Ph.D. Qualifiers Exam – Spring Semester 2020

__MECHANICS OF MATERIALS

EXAM AREA

Assigned Number (DO NOT SIGN YOUR NAME)

✷ Please sign your **name** on the back of this page —

PLEASE NOTE:

You need to correctly answer only 3 out of the 4 problems to receive full credit. In case you attempt all 4 problems, clearly state which 3 problems you want to be graded. If you do not explicitly so indicate, the 3 lowest scores will be used.

Problem #1

Three round alloy bars having the same length L but different shapes are shown in the figure. The first bar has a diameter d over is entire length, the second has a diameter d over a length of x in the middle, and the third has a diameter d over a length of y in the middle. Elsewhere, the second and third bars have diameter $2d$. All three bars are subjected to the same axial load P. The strain-stress curve of the alloy is below.

Use the following numerical data: P = 1400 kN, L = 5 m, d = 80 mm.

(a) If the change in length of bar 1 is 2.5 times of bar 2, find x.

(b) If the strain energy of bar 3 is 30% of that of bar 1, find y.

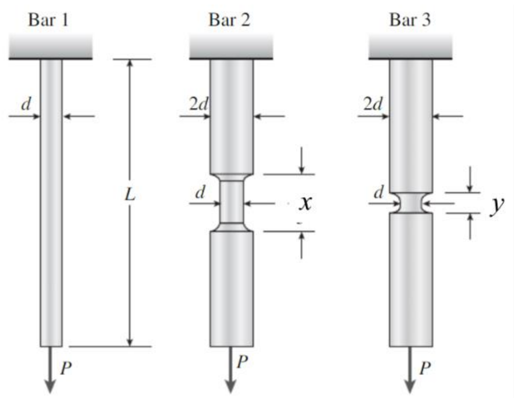

Figure 1a

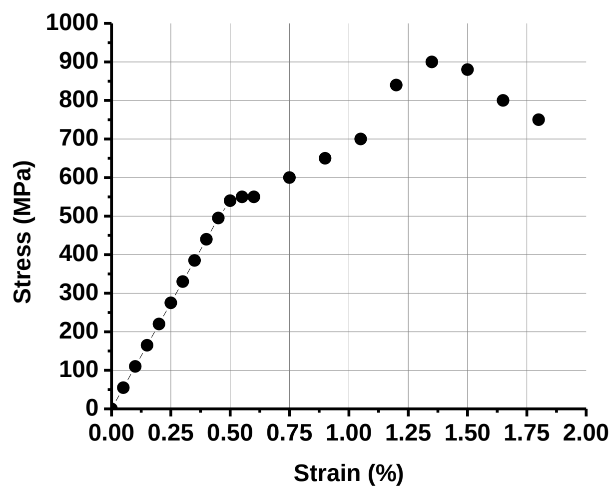

Figure 1b

Problem #2

Please consider a cantilevered aluminum alloy rectangular beam with an applied force, as shown in Figure 2.

(a) When you determine the work done from the external force to the beam, what equation can you develop to describe the external force, displacement, and work done? What are the assumptions you made to get this equation?

(b) Determine the vertical displacement of end *B* of the cantilevered aluminum alloy rectangular beam.
- Assume that there is no energy loss.
- Please consider both shear and bending strain energy.
- Form factor for a rectangular cross section = 1.2, shear modulus = 26 GPa, elastic modulus = 68.9 GPa.

(c) What is the dominant energy that governs the beam deflection?

(d) How can you re-design this beam to make a balance of the share and bending strain energy?

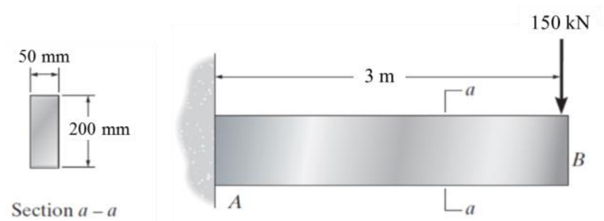

Figure 2

[Formulas]

Strain energy

$$U_i = \frac{N^2L}{2AE} \quad \text{constant axial load}$$

$$U_i = \int_0^L \frac{M^2dx}{2EI} \quad \text{bending moment}$$

$$U_i = \int_0^L \frac{f_sV^2dx}{2GA} \quad \text{transverse shear}$$

$$U_i = \int_0^L \frac{T^2dx}{2GJ} \quad \text{torsional moment}$$

Problem #3

The composite shaft AD is made of a material with shear modulus G and is fixed to rigid walls at A and D. It is subjected to the applied torques T_B and T_C. A hole with diameter d/2 was drilled into the segment AB of the shaft, here denoted segment (1). T_B, T_C, L, G and d are given.

(a) Determine if it is statically determinate problem or indeterminate problem.

(b) Sketch suitable free-body diagrams and relate the internal torques T_1, T_2, and T_3 to the external torque T_B and T_C.

(c) Determine the twist angle at C, indicated as in the figure ϕ_C, as a function of the given values.

(d) Determine the magnitude of the **minimum** shear stress in segment (1) as a function of the given values.

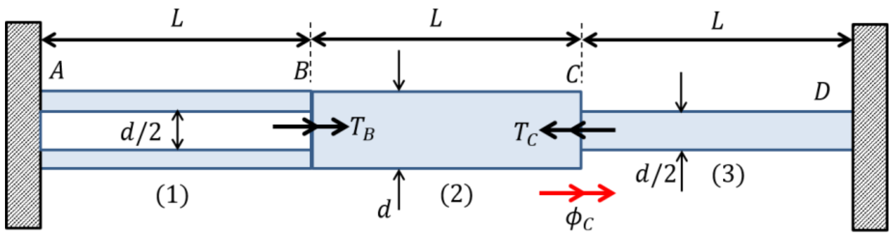

Figure 3

Problem #4

(a) A component is made of a steel for which the critical stress intensity factor for fracture is K_c = 54 MN m$^{-3/2}$. Non-destructive testing by ultrasonic methods shows that the component contains cracks of up to 2*a* = 0.2 mm in length. Laboratory tests show that the crack growth rate under cyclic loading is given by

$$\frac{da}{dN} = A\left(\Delta K\right)^4$$

where $A = 4\times10^{-13}\left(\text{MN m}^{-2}\right)^{-4}\text{ m}^{-1}$. The component is subjected to an alternating stress of range

$$\Delta\sigma = 180\text{ MN m}^{-2}$$

about a mean tensile stress of $\Delta\sigma/2$. Given that $\Delta K = \Delta\sigma\sqrt{\pi a}$, calculate the number of cycles to failure.

(b) An aluminum alloy for an airframe component was tested in the laboratory under an applied stress which varied sinusoidally with time about a mean stress of zero. The alloy failed under a stress range, $\Delta\sigma$, of 280 MN m^{-2} after 10^5 cycles; under a range of 200 MNm^{-2}, the alloy failed after 10^7 cycles.

Assuming that the fatigue behavior of the alloy can be represented by

$$\Delta\sigma\left(N_\text{f}\right)^a = C$$

where *a* and *C* are material constants, find the number of cycles to failure, N_f, for a component subjected to a stress range of 150 MN m^{-2}.